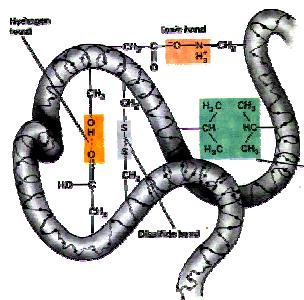
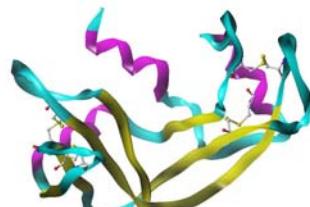


Tertiary Structure of Protein

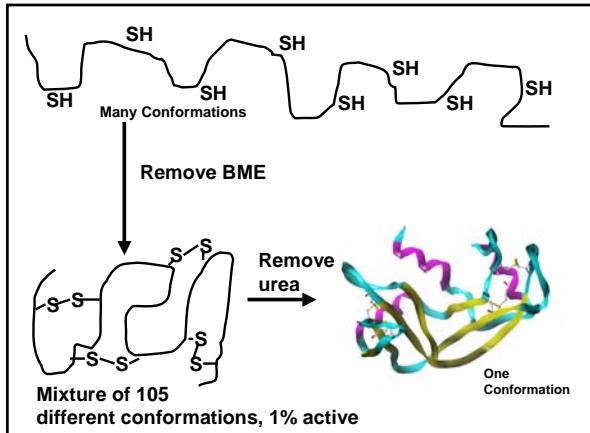
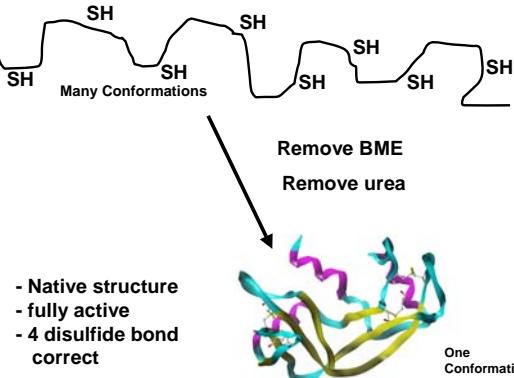
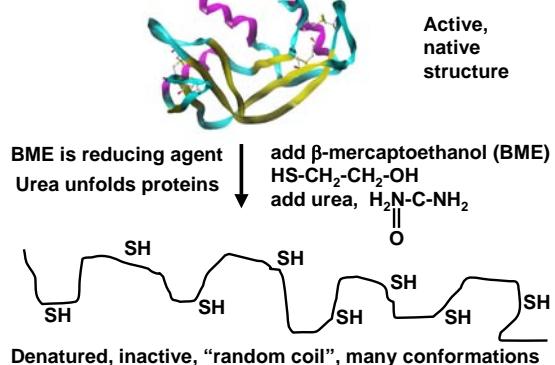
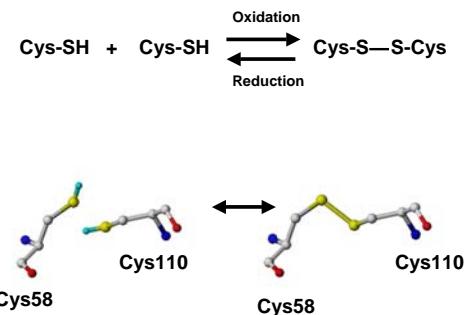


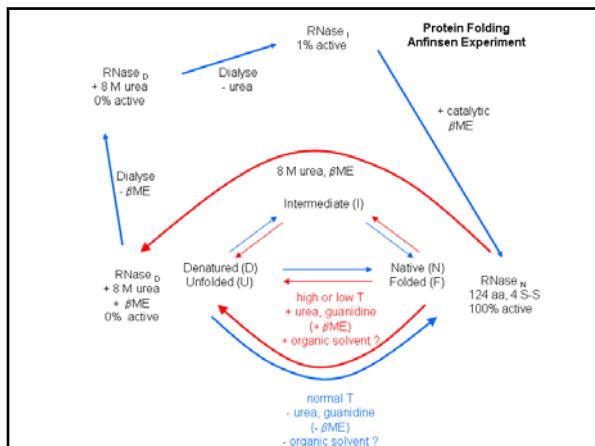
Anfinsen's experiments, late 1950's through 1960's



Ribonuclease, an enzyme involved in cleavage of nucleic acids. Structure has a combination of α and β segments and four disulfide bridges

What are Disulfide Bridges?

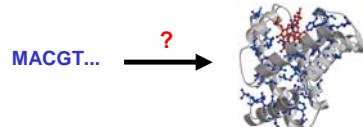




The Protein Folding Problem

Levinthal's paradox – Consider a 100 residue protein. If each residue can take only 3 positions, there are $3^{100} = 5 \times 10^{27}$ possible conformations.

If it takes 10^{-13} s to convert from 1 structure to another, exhaustive search would take 1.6×10^{27} years!



"Given a particular sequence of amino acid residues (primary structure), what will the tertiary/quaternary structure of the resulting protein be?"

Protein Structure Prediction and Protein Folding

Fundamental Questions

Protein Structure Prediction

- What is the structure of this protein?
 - Can be experimentally determined, today we know the structure of ~35,000 proteins
 - Can be predicted for some proteins, usually in ~1 day on today's computers

Protein Folding

- How does this protein form this structure?
 - The process or mechanism of folding
 - Limited experimental characterization
- Why does this protein form **this** structure?
 - Why not some other fold?
 - Why so quickly? -> Levinthal's Paradox: As there are an astronomical number of conformations possible, an unbiased search would take too long for a protein to fold. Yet most proteins fold in less than a second!

Protein Folding: Fast Folders

Time Scale:



- Trp-cage, designed mini-protein (20 aa): 4μs
- β-hairpin of C-terminus of protein G (16 aa) : 6μs
- Engrailed homeodomain (En-HD) (61 aa): ~27μs
- WW domains (38-44 aa): >24μs
- Fe(II) cytochrome b₅₆₂ (106 aa): extrapolated ~5μs
- B domain of protein A (58 aa): extrapolated ~8μs

Structure Prediction Methods



- Secondary structure (only sequence)
- Homology modeling (using related structure)
- Fold recognition
- Ab-initio 3D prediction

Homology Modeling

- Assumes similar (homologous) sequences have very similar tertiary structures
- Basic structural framework is often the same (same secondary structure elements packed in the same way)
- Loop regions differ
 - Wide differences possible, even among closely related proteins

Threading

- Given:
 - sequence of protein P with unknown structure
 - Database of known folds
- Find:
 - Most plausible fold for P
 - Evaluate quality of such arrangement
- Places the residues of unknown P along the backbone of a known structure and determine stability of side chains in that arrangement

Strategies for Protein Structure Prediction

	Comparative Modeling	Fold Recognition	Ab Initio
Method	1. Identify sequence homologs at template 2. Use sequence alignment to generate model 3. Fill in unaligned regions 4. Improve with data	1. Fold classification 2. 3D-Profiles 3. Improves with data	1. Representation 2. Force field 3. Global Optimization 4. Structure at global minimum 5. Can discover new folds
Drawbacks	1. Requires > 25% sequence identity 2. Loops and sidechain conformations are critical	1. Needs good number of proteins in each fold 2. Critically dependent on scoring function	1. Computationally intensive 2. Physical modeling
Resolution	< 3 Å	3 - 7 Å	> 5 Å
Time to Compute	< Day	~ Day	>> Day

Complementarity of the Methods

- X-ray crystallography**- highest resolution structures; faster than NMR
- NMR**- enables widely varying solution conditions; characterization of motions and dynamic, weakly interacting systems
- Computation**- fundamental understanding of structure, dynamics and interactions; models without experiment; very fast

The protein sequence contains all information needed to create a correctly folded protein.

- Many proteins fold spontaneously to their native structure
- Protein folding is relatively fast
- Chaperones speed up folding, but do not alter the structure



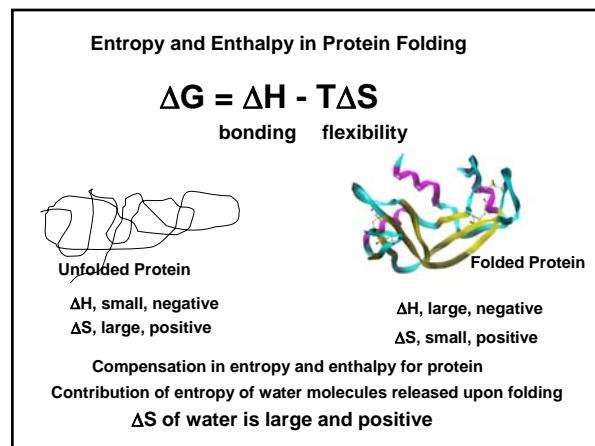
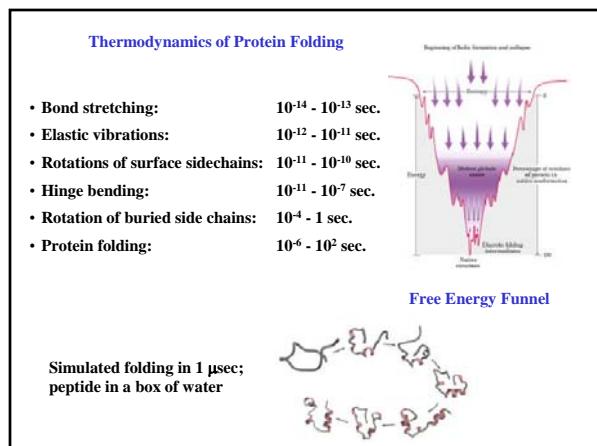
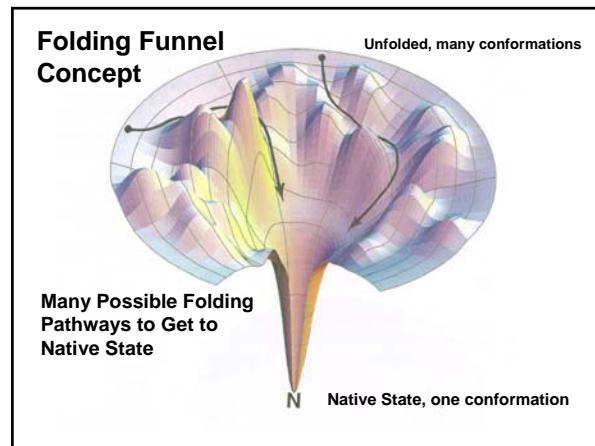
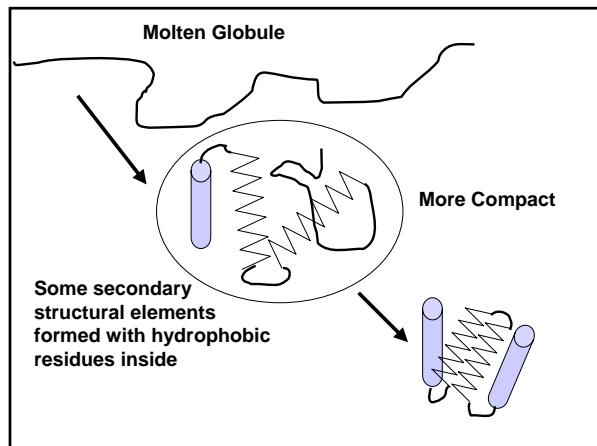
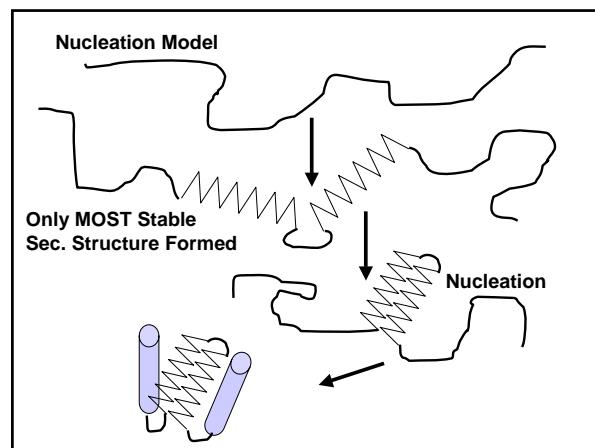
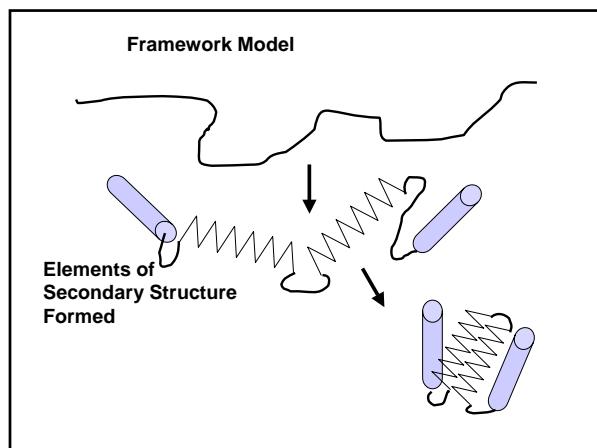
Forces driving protein folding

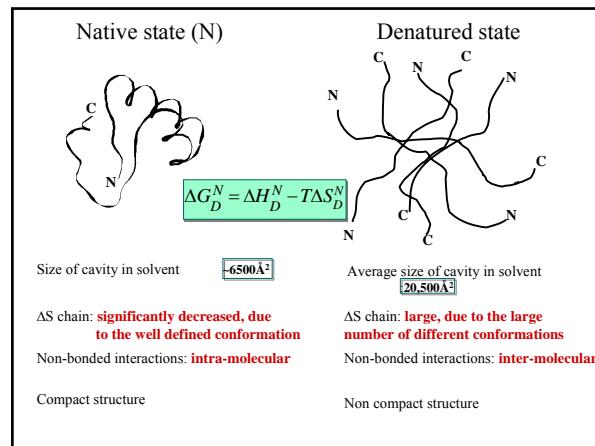
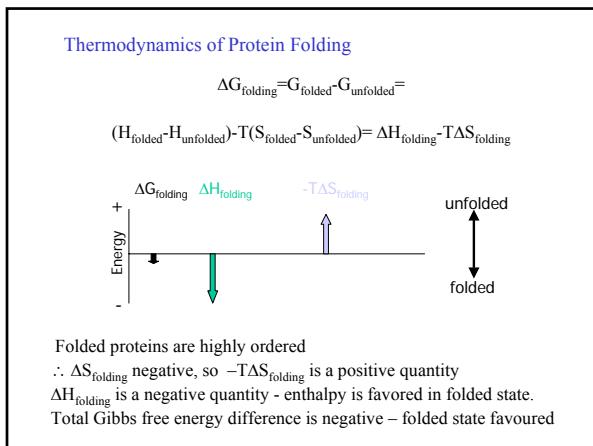
- It is believed that *hydrophobic collapse* is a key driving force for protein folding
 - Hydrophobic core
 - Polar surface interacting with solvent
- Minimum volume (no cavities)
- Disulfide bond formation stabilizes
- Hydrogen bonds
- Polar and electrostatic interactions

Native state is typically only 5 to 10 kcal/mole more stable than the unfolded form

Four models that could account for the rapid rate of protein folding during biological protein synthesis.

- The Framework Model
- The Nucleation Model
- The “Molten Globule” Model
- “Folding Funnel”





Factors that disrupt the Native state

- 1) **ELECTROLYTE ADDITION**
 - interference with the colloid state
- 2) **INSOLUBLE SALT FORMATION**
 - Protein+Trichloroacetate
- 3) **ORGANIC SOLVENTS**
 - ETHANOL - interferes with the dielectric constant
- 4) **HEAT DENATURATION**
 - more energy in system (bonds break)
- 5) **pH**
 - destroys charge
 - destroys ability to interact with water
- 6) **DESTRUCTION OF HYDROGEN BONDING**
 - UREA - known H-bond disrupter

Thermodynamic Description of Protein Folding

The native and unfolded states are in equilibrium, the folding reaction can be quantified in terms of thermodynamics.

The equilibrium $(N \leftrightarrow U)$ between the native (N) and unfolded (U) states is defined by the equilibrium constant, K , as:

$$K = [U]/[N] = K_U$$

The difference in Gibbs free energy (ΔG) between the unfolded and native states is then:

$$\Delta G = -RT \ln K$$

For K_u , a positive ΔG indicates that the native state is more stable.

The free energy is composed of both enthalpic and entropic contributions:
 $\Delta G = \Delta H - T \Delta S$

where ΔH and ΔS are the enthalpy and entropy change, respectively, upon unfolding.

Thermal Unfolding
 Since ΔH and ΔS are strongly temperature-dependent, ΔG is better expressed as:

$$\Delta G = \Delta H_i + \Delta C_p (T - T_i) - T [\Delta S_i + \Delta C_p \ln(T/T_i)]$$

where the subscript "i" indicates the value of ΔH and ΔS at a reference temperature, T_i , and ΔC_p is the specific heat or heat-capacity change.

Most proteins denature reversibly allowing thermodynamic analysis.

Factors that disrupt the Native state

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Solving Protein Structures

Only 2 kinds of techniques allow one to get atomic resolution pictures of macromolecules

- X-ray Crystallography (first applied in 1961 - Kendrew & Perutz)
- NMR Spectroscopy (first applied in 1983 - Ernst & Wuthrich)

- Structure \longleftrightarrow Function Protein 3D Structure Prediction
- Structure \longleftrightarrow Mechanism
- Structure \longleftrightarrow Origins/Evolution
- Structure-based Drug Design
- Solving the Protein Folding Problem

Ab Initio Prediction

- Predicting the 3D structure without any "prior knowledge"
- Used when homology modelling or threading have failed (no homologues are evident)
- Equivalent to solving the "Protein Folding Problem"
- Still a research problem

Ab Initio Folding

Two Central Problems

- Sampling conformational space (10^{100})
- The energy minimum problem

The Sampling Problem (Solutions)

- Lattice models, off-lattice models, simplified chain methods

The Energy Problem (Solutions)

- Threading energies, packing assessment, topology assessment

Problems in Protein Folding

Two key questions:

- Evaluation - how can we tell a correctly-folded protein from an incorrectly folded protein?
 - H-bonds, electrostatics, hydrophobic effect, etc.
 - Derive a function, see how well it does on "real" proteins
- Optimization - once we get an evaluation function, can we optimize it?
 - Simulated annealing/Monte Carlo

Interaction	Approx. bond strength in kJ/mole
Covalent bonds	> 200 (ranging up to 900)
Ionic	20-40
Hydrogen bond	-5-20
Hydrophobic	-8
van der Waals	-4

Evaluation of Protein Folds

- Empirical potential functions
 - Residue-based: spatial relationships among residues
 - Stereochemistry-based: molecular interactions (covalent, electrostatic, etc.) with coefficients
- Ab-initio potential functions
- Procheck, etc.
- Full molecular dynamics
 - Very computationally expensive

AMBER (Assisted Model Building with Energy Refinement) force field

$$E_{total} = \sum_{bonds} K_r (r - r_{eq})^2 + \sum_{angles} K_\theta (\theta - \theta_{eq})^2 + \sum_{dihedrals} \frac{3}{2} V_n [1 + \cos(n\omega)] + \sum_{i < j} \left(\frac{a_{ij}}{r_{ij}^{12}} - \frac{b_{ij}}{r_{ij}^6} \right) + \sum_{i < j} \frac{q_i q_j}{\epsilon r_{ij}}$$

Polypeptides

- Represented by a range of approaches or approximations including:
 - all atom representations in cartesian space
 - all atom representations in dihedral space
 - simplified atomic versions in dihedral space
 - tube/cylinder/ribbon representations
 - **lattice models**

Lattice Models

- The “hydrophobic zipper” effect:

Ken Dill ~ 1997

Scoring Lattice Models

- H/P model scoring: count noncovalent hydrophobic interactions.

Sometimes:
– Penalize for buried polar or surface hydrophobic residues

Fold Optimization

- Simple lattice models (H/P-models)
 - Two types of residues: hydrophobic and polar
 - 2-D or 3-D lattice
 - The only force is hydrophobic collapse
 - Score = number of H-H contacts

A Simple 2D Lattice

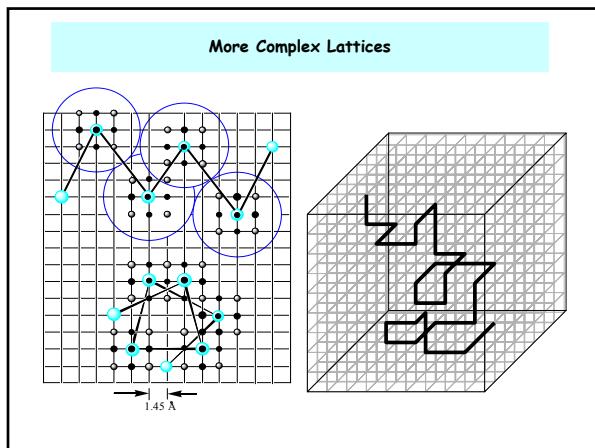
3.5 Å

Lattice Folding

Lattice Algorithm

- Build a “ $n \times m$ ” matrix (a 2D array)
- Choose an arbitrary point as your N terminal residue (start residue)
- Add or subtract “1” from the x or y position of the start residue
- Check to see if the new point (residue) is off the lattice or is already occupied
- Evaluate the energy
- Go to step 3 and repeat until done

- If **Red** is near empty space $E = E+1$
- If **Blue** is near empty space $E = E-1$
- If **Red** is near another **Red** $E = E-1$
- If **Blue** is near another **Blue** $E = E+0$
- If **Blue** is near **Red** $E = E+0$

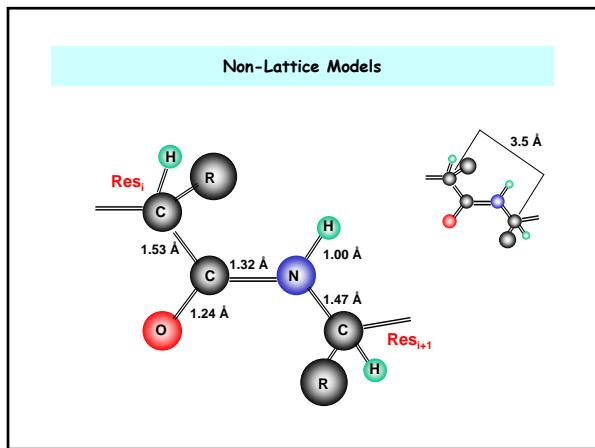


What can we do with lattice models?

- For smaller polypeptides, exhaustive search can be used
 - Looking at the "best" fold, even in such a simple model, can teach us interesting things about the protein folding process

More realistic models

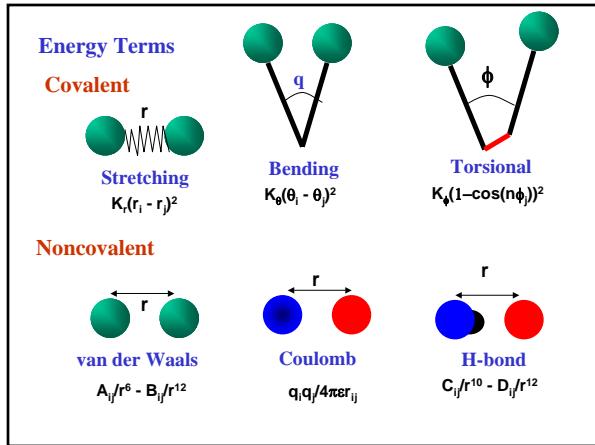
- Higher resolution lattices (45° lattice, etc.)
- Off-lattice models
 - Local moves
 - Optimization/search methods and ϕ/ψ representations
 - Greedy search
 - Graph theoretical methods
 - Monte Carlo, simulated annealing, etc.



Non-Lattice Models

- With a more realistic off-lattice model, we need a better *energy function* to evaluate a conformation (fold).
- Theoretical force field:

$$\Delta G = \Delta G_{\text{van der Waals}} + \Delta G_{\text{h-bonds}} + \Delta G_{\text{solvent}} + \Delta G_{\text{coulomb}}$$
- Empirical force fields



Bonding Terms: bond stretch

- Most often Harmonic

$$V_{\text{bond}} = \sum_{\text{bonds}} \frac{1}{2} k_r (r - r_0)^2$$
- Morse Potential for dissociation studies

$$V_{\text{Morse}} = \sum_{\text{bonds}} D [e^{-a(r-r_0)} - 1]^2 - D$$

Two new parameters:
 D: dissociation energy
 a: width of the potential well

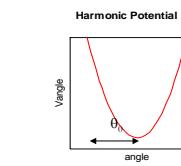
Bonding Terms: angle bending

- Most often Harmonic

$$V_{\text{angle}} = \sum_{\text{angles}} \frac{1}{2} k_{\theta} (\theta - \theta_0)^2$$

- CHARMM force field's Urey-Bradley angle term:

$$V_{UB} = \sum_{UB} \frac{1}{2} k_{UB} (s - s_0)^2$$



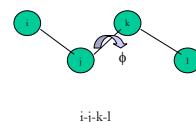
This UB term is only found in CHARMM force field to optimize the fit to vibrational spectra.
s: the 1,3-distance.

Mackerell et al. J. Phys. Chem. B 102, 3586, 1998

Bonding Terms: Torsions

- Torsion energy: rotation about a bond (dihedral angles)

$$U_{\text{torsion}} = \sum_{\text{torsions}} \frac{V_n}{2} [1 + \cos(n\phi - \delta)]$$



V_n: force constant
n: periodicity of the angle (determines how many peaks and wells in the potential, often from 1-6)
δ: phase of the angle (often 0° or 180°)

Bonding Terms: Improper Torsions

- Improper torsion is not a regular torsion angle. It is used to describe the energy of out-of-plane motions. It is often necessary for planar groups, such as sp² hybridized carbons in carbonyl groups and in aromatic rings, because the normal torsion terms described above is not sufficient to maintain the planarity ($\omega \sim 0$).

$$U_{\text{improper}} = \sum_{\text{improper}} \frac{V_2}{2} [1 + \cos(2\omega - 180^\circ)]$$

or

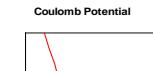
$$U_{\text{improper}} = \sum_{\text{improper}} \frac{k_w}{2} (\omega - \omega_0)^2$$



Non-bonded Terms

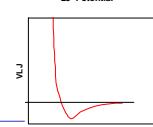
- Electrostatic interactions (Coulomb's Law)

$$V_{\text{elec}} = \frac{1}{4\pi\epsilon} \sum_{i < j} \frac{q_i q_j}{r_{ij}}$$



- Lennard-Jones interactions

$$V_{\text{LJ}} = \sum_{i < j} 4\epsilon_{ij} \left[\frac{\sigma_{ij}^{12}}{r_{ij}^{12}} - \frac{\sigma_{ij}^6}{r_{ij}^6} \right]$$

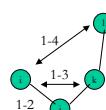


- Combination Rules for LJ

$$\epsilon_{ij} = \sqrt{\epsilon_i \epsilon_j} \quad \sigma_{ij} = \frac{1}{2} (\sigma_i + \sigma_j) \quad \sigma_{ij} = \sqrt{\sigma_i \sigma_j}$$

1-4 Non-bonded Interactions

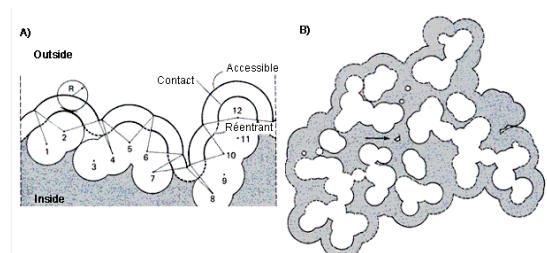
- Non-bonded exclusions
 - 1-2 and 1-3 interactions excluded
 - 1-4 interactions partially excluded
- 1-4 interaction scalings
 - OPLSAA scales by 0.5 for both electrostatic and LJ
 - AMBER94 scales 0.5 for LJ and 1/1.2 for electrostatic interaction
 - CHARMM22 has special 1,4-terms

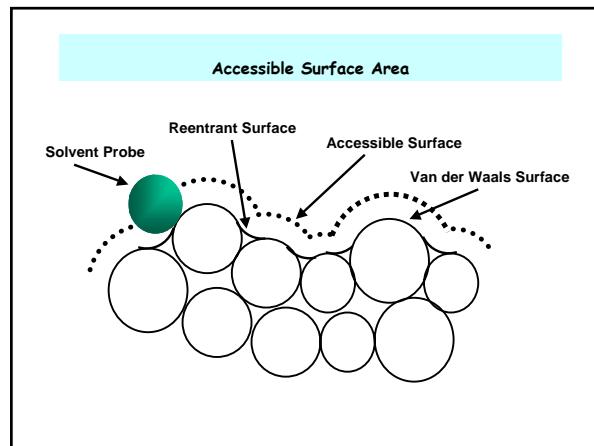
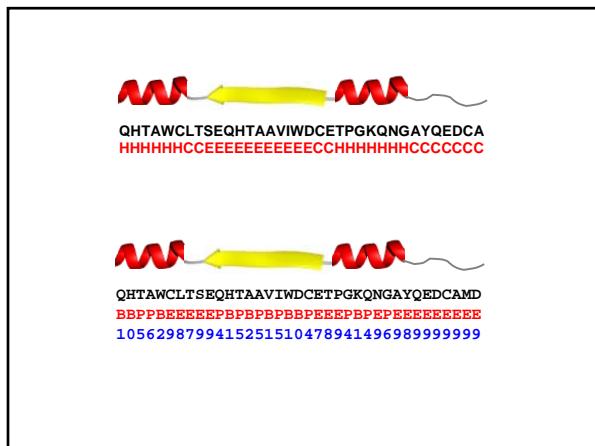
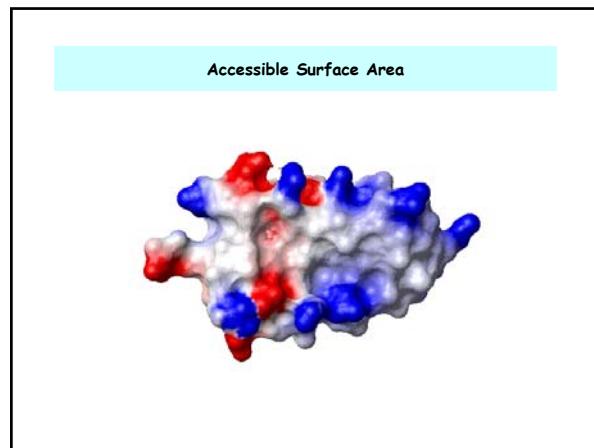
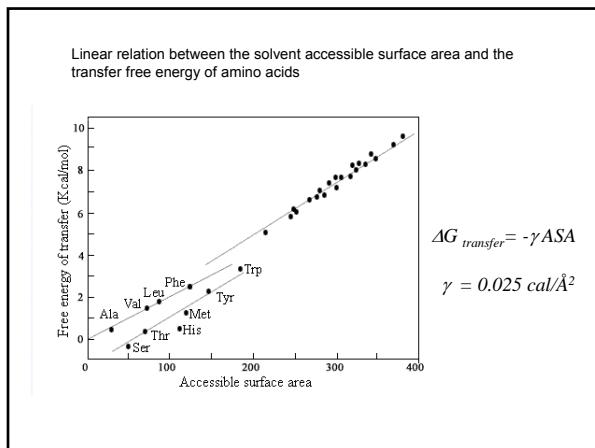


Even though they are non-bonded interactions, 1-4 terms are often calculated along with bonded terms.

The hydrophobic effect

The free energy gain from burying a hydrophobic group is proportional to the surface area buried





Accessible Surface Area Calculations

- DSSP - Database of Secondary Structures for Proteins (swift.embl-heidelberg.de/dssp)
- Connolly Molecular Surface Home Page
 - <http://www.biohedron.com/>
- Naccess Home Page
 - <http://sjh.bi.umist.ac.uk/naccess.html>
- ASA Parallelization
 - <http://cmag.cit.nih.gov/Asa.htm>
- Protein Structure Database
 - <http://www.psc.edu/biomed/pages/research/PSdb/>

Force Fields: Typical Energy Functions

$$\begin{aligned}
 U = & \sum_{bonds} \frac{1}{2} k_r (r - r_0)^2 && \text{Bond stretches} \\
 & + \sum_{angles} \frac{1}{2} k_\theta (\theta - \theta_0)^2 && \text{Angle bending} \\
 & + \sum_{torsions} \frac{V_n}{2} [1 + \cos(n\phi - \delta)] && \text{Torsional rotation} \\
 & + \sum_{improper} V(\text{improper torsion}) && \text{Improper torsion (sp2)} \\
 & + \sum_{elec} \frac{q_i q_j}{r_{ij}} && \text{Electrostatic interaction} \\
 & + \sum_{IJ} \left[\frac{A_{ij}}{r_{ij}^{12}} - \frac{B_{ij}}{r_{ij}^6} \right] && \text{Lennard-Jones interaction}
 \end{aligned}$$

Which Force Field to Use?

- Most popular force fields: CHARMM, AMBER and OPLSAA
- OPLSAA(2000): Probably the best available force field for condensed-phase simulation of peptides. Work to develop parameterization that will include broader classes of drug-like molecules is ongoing. GB/SA solvation energies are good.
- MMFF: An excellent force field for biopolymers and many drug-like organic molecules that do not have parameters in other force fields.
- AMBER*/OPLS*: Good force fields for biopolymers and carbohydrates; many parameters were added in MacroModel which extend the scope of this force field to a number of important organic functional groups. GB/SA solvation energies range from moderate (AMBER*) to good (OPLS*).
- AMBER94: An excellent force field for proteins and nucleic acids. However, there are no extensions for non-standard residues or organic molecules, also there is a alpha-helix tendency for proteins. AMBER99 fixes this helix problem to some degree, but not completely.
- MM2*MM3*: Excellent force fields for hydrocarbons and molecules with single or remotely spaced functional groups. GB/SA solvation energies tend to be poor relative to those calculated with other force fields.
- CHARMM22: Good general purpose force field for proteins and nucleic acids. A bit weak for drug-like organic molecules.
- GROMOS96: Good general purpose force field for proteins, particularly good for free energy perturbations due to soft-core potentials. Weak for reproducing solvation free energies of organic molecules and small peptides.

http://www.schrodinger.com/docs/mm7.1/html/faqs/which_ffield.html

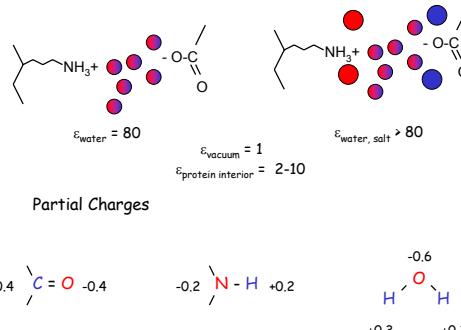
Force Field Parameterization

- Equilibrium bond distances and angles: X-ray crystallography
- Bond and angle force constants: vibrational spectra, normal mode calculations with QM
- Dihedral angle parameters: difficult to measure directly experimentally; fit to QM calculations for rotations around a bond with other motions fixed
- Atom charges: fit to experimental liquid properties, ESP charge fitting to reproduce electrostatic potentials of high level QM, X-ray crystallographic electron density
- Lennard-Jones parameters: often most difficult to determine, fit to experimental liquid properties, intermolecular energy fitting

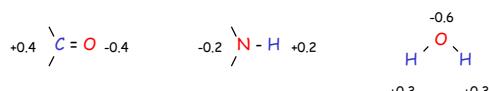
Applications

- NMR or X-ray structure refinement
- Protein structure prediction
- Protein folding kinetics and mechanics
- Conformational dynamics
- Global optimization
- DNA/RNA simulations
- Membrane proteins/lipid layers simulations

Dielectric constant

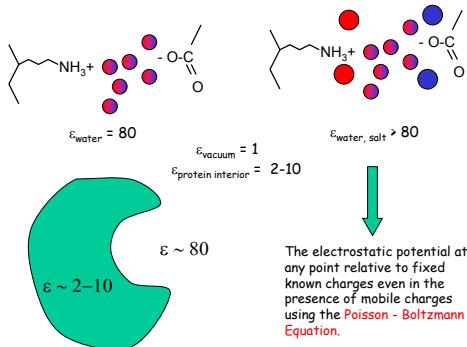


Partial Charges



If you know the position of every partial charge (including water), you do not need a dielectric constant.

Dielectric constant



The electrostatic potential at any point relative to fixed known charges even in the presence of mobile charges using the Poisson - Boltzmann Equation.

Dipole - Monopole Interactions

$$U = \sum U(n) = U_1 + U_2$$

monopole q_0

dipole q

$U = 1/4\pi\epsilon_0 (q/r_1 - q/r_2) = q/4\pi\epsilon_0 (r_1 - r_2/r_1 r_2)$

if $r \gg a$ then $r_2 - r_1 \sim a \cos \theta$ and $r_1 r_2 = r^2$

$U = qa/4\pi\epsilon_0 (\cos \theta / r^2)$

U is a function of θ and r . If you rotate around the dipole axis, there is no change in the value of U

Dipole - Dipole Interactions

μ = dipole moment = Zd

water = 1.85 D

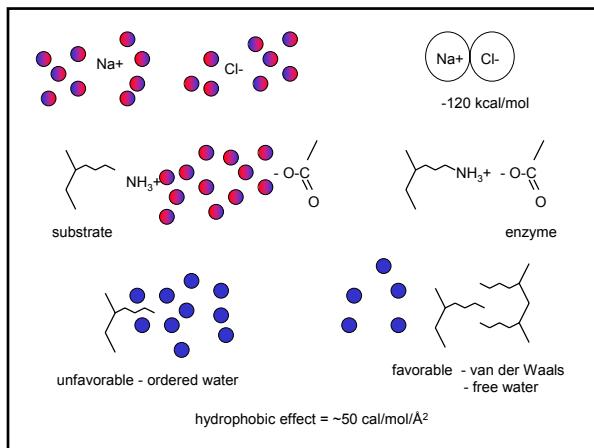
peptide bond = 3.5 D

retinal = 15 D

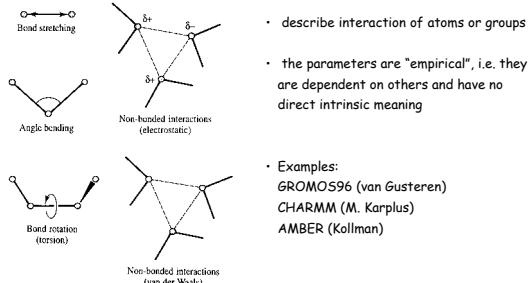
$E = -2\mu_a \mu_b / \epsilon r^3$

$E = -\mu_a \mu_b / \epsilon r^3$

Interaction energy is dependent on orientation and distance



Empirical Force Fields and Molecular Mechanics



Example for a (very) simple Force Field:

$$\begin{aligned}
 V = & \sum_{bonds} \frac{k_i}{2} (l_i - l_{i,0})^2 \\
 & + \sum_{angles} \frac{k_i}{2} (\theta_i - \theta_{i,0})^2 \\
 & + \sum_{torsions} \frac{V_n}{2} (1 + \cos(n\omega - \gamma)) \\
 & + \sum_{i=1}^N \sum_{j=i+1}^N \left(4\pi\epsilon_{ij} \left[\left(\frac{\sigma_{ij}}{r_{ij}} \right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}} \right)^6 \right] + \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}} \right)
 \end{aligned}$$

Complete Energy Function:

$$\begin{aligned}
 H = & \sum_{atoms} \frac{p^2}{2m} + \sum_{bond-stretch} \frac{1}{2} k_r (r - r_{eq})^2 + \sum_{bond-angle-bending} \frac{1}{2} k_\theta (\theta - \theta_{eq})^2 + \\
 & \sum_{bond-rotation} \frac{V_n}{2} [1 + \cos(n\phi - \gamma)] + \sum_{S-bond} [V_0 (1 - e^{-a(r - r_0)})^2 - V_0] + \\
 & \sum_{bond-bond} [V_0 (1 - e^{-a(r - r_0)})^2 - V_0] + \sum_{non-bonded} \left[\frac{A_{ij}}{r_{ij}^{12}} - \frac{B_{ij}}{r_{ij}^6} + \frac{q_i q_j}{\epsilon_{ij} r_{ij}} \right]
 \end{aligned}$$

AMBER (Assisted Model Building with Energy Refinement) force field

$$\begin{aligned}
 E_{total} = & \sum_{bonds} K_r (r - r_{eq})^2 + \sum_{angles} K_\theta (\theta - \theta_{eq})^2 + \sum_{dihedrals} \frac{3}{2} \sum_{i=1}^3 V_n [1 + \cos(n\omega)] \\
 & + \sum_{atoms} \left(\frac{a_{ij}}{r_{ij}^{12}} - \frac{b_{ij}}{r_{ij}^6} \right) + \sum_{i < j} \frac{q_i q_j}{\epsilon_{ij} r_{ij}}
 \end{aligned}$$

Sources of force parameters:

Bonds, VdW, Electrostatic (for amino acids, nucleotides only):

- AMBER: J. Am. Chem. Soc. 117, 5179-5197
- CHARMM: J. Comp. Chem. 4, 187-217

H-bonds (Morse potential):

- Nucleic Acids Res. 20, 415-419.
- Biophys. J. 66, 820-826

Electrostatic parameters of organic molecules need to be computed individually by using special software (such as Gaussian)

Average energy scale for various interactions:

Energy Term	Scale (kcal/mol)
Bond stretching	100
Angle Bending	10
Torsion	1
Hydrogen Bond	2
Electrostatic interaction	0.5
Van der Waals	0.1

Concept of energy scale is Important for molecular modeling

Average bond energies in units of kJ/mol
(1kJ/mol=0.239 kcal/mol):

A. Single bonds:

	S	P	O	N	C	H
H	339	318	463	389	414	436
C	259	264	351	293	347	
N			209	201	159	
O			351	138		
P	230	213				
S	213					

B. Multiple bonds:

N=N	418	C=C	611
N≡N	946	C≡C	837
C=N	615	C=O (in CO ₂)	803
C≡N	891	C=O (as in H ₂ C=O)	745
O=O	498	C=O	1075

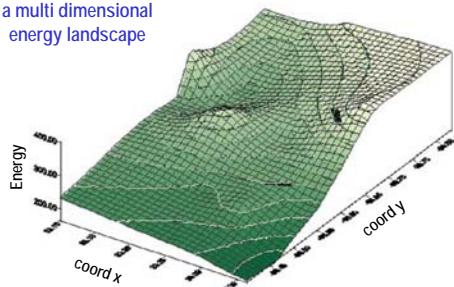
Energy Minimization

- $E = f(x)$
- E is a function of coordinates either cartesian or internal
- At minimum the first derivatives are zero and the second derivatives are all positive

$$\frac{dE}{dx_i} = 0$$

$$\frac{d^2E}{dx_i^2} \rightarrow 0$$

Potential Energy Surface (PES)
a multi dimensional energy landscape



- Systematic Searching
 - explore the whole PES
- Stochastic Searching
 - find "all" low energy minima by generating starting conformation with random changes of rotatable dihedral angles (sometimes combined with random perturbation of the Cartesian coordinates) followed by minimization
- Monte Carlo Simulations
 - generate a Boltzmann distributed ensemble of conformations, can estimate macroscopic thermodynamic properties
- Molecular Dynamics
 - Simulates the time dependent motion of the molecular system, can estimate macroscopic thermodynamic properties
- Simulated Annealing
 - Playing with the temperature (T) in either MD or MC simulations to speed up search for low energy minima
- Distance geometry
 - method for generating conformations that satisfy experimental constraints

Systematic Searching

$$\text{Number of Conformers} = \left(\frac{360}{x}\right)^n$$

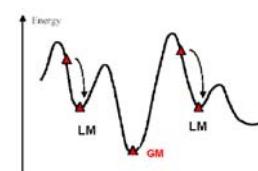
where x is the angle increment
 n is the number of rotatable bonds

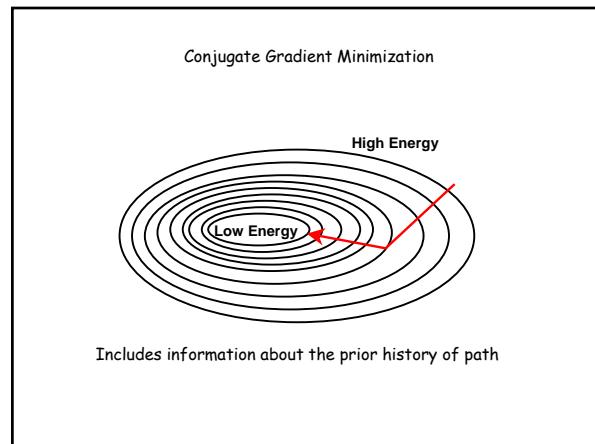
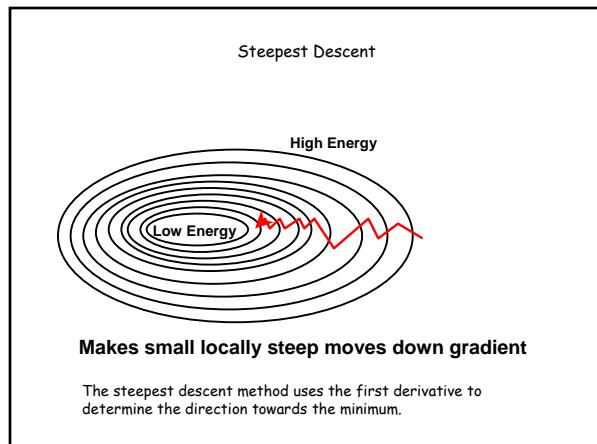
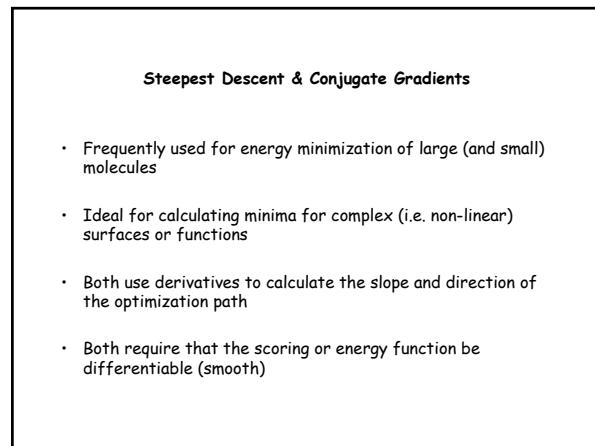
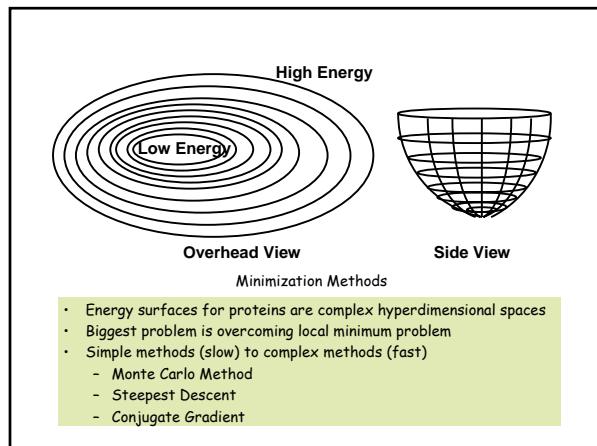
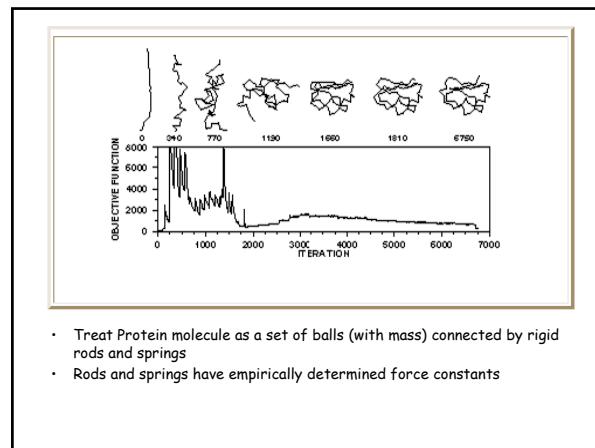
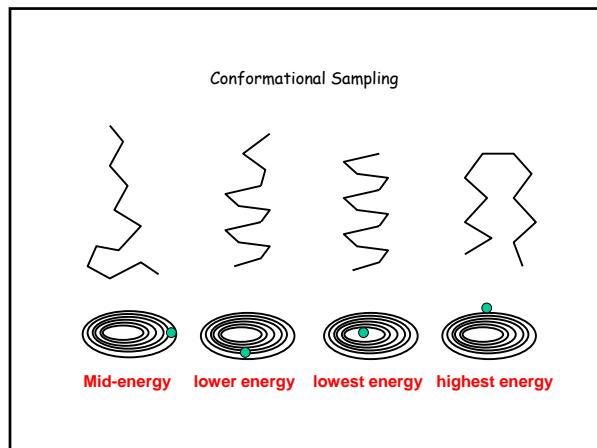
The exhaustive nature of the search is a combinatorial problem

Rotatable Bonds, n	Increment, x	Conformers
3	30°	1728
3	15°	13824
3	7.5°	110592
4	30°	20736
5	30°	248832
6	30°	2985984

Molecular Mechanics - Energy Minimization

- The energy of the system is minimized. The system tries to relax
- Typically, the system relaxes to a local minimum (LM).

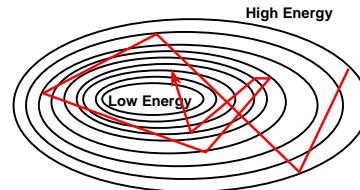




Monte Carlo Algorithm

- Generate a conformation or alignment (a state)
- Calculate that state's energy or "score"
- If that state's energy is less than the previous state accept that state and go back to step 1
- If that state's energy is greater than the previous state accept it if a randomly chosen number is $< e^{-E/kT}$ where E is the state energy otherwise reject it
- Go back to step 1 and repeat until done

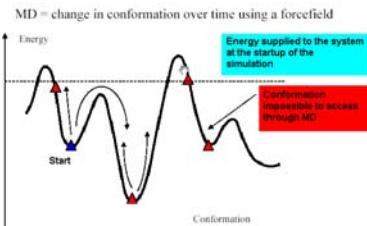
Monte Carlo Minimization



Performs a progressive or directed random search

Molecular Dynamics (MD)

In molecular dynamics, energy is supplied to the system, typically using a constant temperature (i.e. constant average kinetic energy).



Molecular Dynamics (MD)

- Use Newtonian mechanics to calculate the net force and acceleration experienced by each atom.
- Each atom i is treated as a point with mass m_i and fixed charge q_i
- Determine the force F_i on each atom:

$$\vec{F}_i = m_i \frac{d^2 \vec{r}_i}{dt^2} = -\vec{\nabla} V(\vec{R})$$

- Use positions and accelerations at time t (and positions from $t - \delta t$) to calculate new positions at time $t + \delta t$

Initial velocities (v_i)

using the Boltzmann distribution at the given temperature

$$v_i = (m_i/2\pi kT)^{1/2} \exp(-m_i v_i^2/2kT)$$

Molecular dynamics (MD) simulations

- A deterministic method based on the solution of Newton's equation of motion

$$\vec{F}_i = m_i \vec{a}_i$$

for the i th particle: the acceleration at each step is calculated from the negative gradient of the overall potential, using

$$\vec{F}_i = -\nabla V_i = -\nabla V$$

In molecular dynamics forces are derived from a potential energy function V , which depend on the particle coordinates:

$$\vec{F}_i = -\nabla V(\vec{r}_1, \dots, \vec{r}_N)$$

The problem of modelling a material can therefore be restated as that of finding a potential function for that material.

$$V(\vec{r}_1, \dots, \vec{r}_N)$$

Molecular dynamics (MD) simulations

$V_i = \sum_j (energies \text{ of interactions between } i \text{ and all other residues } k \text{ located within a cutoff distance of } R_c \text{ from } i)$

- Derivative of V with respect to the position vector $\vec{r}_i = (x_i, y_i, z_i)^T$ at each step

$$\begin{aligned} a_{xi} &\sim -\partial V / \partial x_i \\ a_{yi} &\sim -\partial V / \partial y_i \\ a_{zi} &\sim -\partial V / \partial z_i \end{aligned}$$

Non-Bonded Interaction Potentials

- Electrostatic interactions of the form $E_{ik}(\text{es}) = q_i q_k / r_{ik}$
- Van der Waals interactions $E_{ij}(\text{vdW}) = -a_{ik}/r_{ik}^6 + b_{ik}/r_{ik}^{12}$

Bonded Interaction Potentials

- Bond stretching $E_i(\text{bs}) = (k_{bs}/2) (l_i - l_i^0)^2$
- Bond angle distortion $E_i(\text{bad}) = (k_\theta/2) (\theta_i - \theta_i^0)^2$
- Bond torsional rotation $E_i(\text{tor}) = (k_\phi/2) f(\cos\phi_i)$

Molecular dynamics (MD) simulations

The Verlet algorithm

The most widely used method of integrating the equations of motion is that initially adopted by Verlet [1967]. The method is based on positions $r(t)$, accelerations $a(t)$, and the positions $r(t-\delta t)$ from the previous step.

The equation for advancing the positions reads as

$$r(t+\delta t) = 2r(t) - r(t-\delta t) + \delta t^2 a(t)$$

The velocities do not appear at all. They have been eliminated by addition of the equations obtained by Taylor expansion about $r(t)$:

$$r(t+\delta t) = r(t) + \delta t v(t) + (1/2) \delta t^2 a(t) + \dots$$

$$r(t-\delta t) = r(t) - \delta t v(t) + (1/2) \delta t^2 a(t) -$$

The velocities are not needed to compute the trajectories, but they are useful for estimating the kinetic energy (and hence the total energy). They may be obtained from the formula

$$v(t) = [r(t+\delta t) - r(t-\delta t)]/2\delta t$$

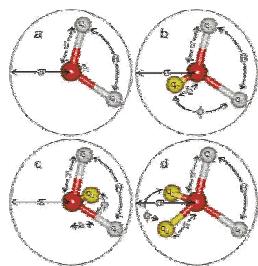
Trajectory file: During molecular dynamics (and energy minimization) the coordinates (and velocities) are saved at regular intervals. Such a file is called a trajectory file.

Water Models

A recent review listed 46 distinct models, so indirectly indicating their lack of success in quantitatively reproducing the properties of real water.

They may, however, offer useful insight into water's behavior.

Models types a, b and c are all planar whereas type d is almost tetrahedral



Implicit Solvent Models

Water molecules are not included as molecules, but represented by an extra potential on the solvent accessible surface.

• only 50% slower than vacuum calculations

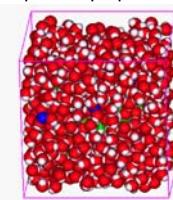
• ~10 times faster than explicit water MD

Explicit Solvent Models

Water molecules are explicitly included as individual molecules.

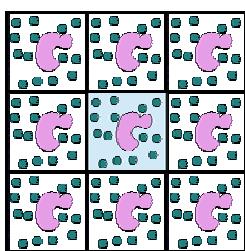
• Force Fields for water molecules are not trivial ...

• Computationally expensive ...



Periodic Boundary Conditions (PBC)

- Periodic boundary conditions are used to simulate solvated systems or crystals.
- In solvated systems, PBC prevents that the solvent "evaporates *in silico*"



Building peptides using Z matrices

	distance	angle	dihedral	connectivity
N	0.0000 0	0.0000 0	0.0000 0	0 0 0
H	1.0200 1	0.0000 0	0.0000 0	1 0 0
H	1.0200 1	104.5368 1	0.0000 0	1 2 0
H	1.0200 1	104.5368 1	109.5796 1	1 2 3
0 (end of file)				

(1 means optimize, 0 means keep constant, -1 means vary according to a designated pattern)

PDB Representation

HETATM	1	C	1	-1.129	1.281	-0.000
HETATM	2	C	2	-2.558	1.772	-0.000
HETATM	3	C	3	-3.519	0.606	-0.000
HETATM	4	H	4	-0.596	1.637	0.890
HETATM	5	H	5	-0.596	1.637	-0.890
HETATM	6	H	6	-2.733	2.392	0.890
HETATM	7	H	7	-2.733	2.392	-0.890
HETATM	8	H	8	-4.558	0.952	0.000
HETATM	9	H	9	-3.359	-0.017	-0.890
HETATM	10	H	10	-3.359	-0.017	0.890
HETATM	11	H	11	-1.110	0.183	-0.000

(blue indicates data columns not utilized/recognized by all software)

Atom types (AMBER)

Table I. List of Atom Types^a

Angle Parameters

PDB Representation contd.

```

CONECT 1 2 4 5 11
CONECT 2 1 3 6 7
CONECT 3 2 8 9 10
CONECT 4 1
CONECT 5 1
CONECT 6 2
CONECT 7 2
CONECT 8 3
CONECT 9 3
CONECT 10 3
CONECT 11 1
END

```

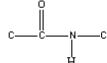
248

Bond Parameters

Bond Parameters							
bond	K^b	r_m^b	bond	K^b	r_m^b	bond	K^b
C-CA	469.0	1.409	CA-RA	367.0	1.080	CH-CA	367.0
C-CB	447.0	1.419	CA-N2	481.0	1.340	CM-N*	448.0
C-CM	410.0	1.444	CA-TD	470.0	1.385	NC-CA	428.0
C-CT	317.0	1.522	CA-NC	485.0	1.339	CO-HS	367.0
C-N	490.0	1.335	CB-CD	520.0	1.370	CH-NC	502.0
C-N*	434.0	1.383	CB-CD	447.0	1.419	CR-HS	367.0
C-NA	418.0	1.388	CB-N*	436.0	1.374	CR-NA	477.0
C-NC	457.0	1.258	CB-N*	436.0	1.374	CR-NB	409.0
C-NC	370.0	1.492	CC-CD	462.0	1.354	CT-CH	310.0
C-NC	656.0	1.250	CC-CT	317.0	1.504	CT-HI	367.0
C-OH	450.0	1.364	CC-CW	512.0	1.375	CT-HI	340.0
C-CB	388.0	1.459	CC-CW	518.0	1.371	CH-CH	340.0
C-CT	317.0	1.495	CC-NA	422.0	1.385	CH-CH	340.0
C-CW	546.0	1.352	CC-NB	410.0	1.394	CH-TC	340.0
C-TC	367.0	1.300	CH-TC	367.0	1.300	CH-TP	310.0
C-TC	367.0	1.300	CH-TP	367.0	1.300	CH-TP	310.0
C-CB	469.0	1.404	CH-TP	367.0	1.371	CH-TP	337.0
C-CM	427.0	1.433	CM-CW	549.0	1.350	CH-TP	337.0
C-CM	486.0	1.400	CM-CT	317.0	1.510	CH-TP	367.0
C-CT	317.0	1.510	CH-DS	367.0	1.080	CH-TP	320.0
C-DS	367.0	1.080	CH-DS	367.0	1.080	CH-TP	320.0

Torsion Parameters

Improper Torsions

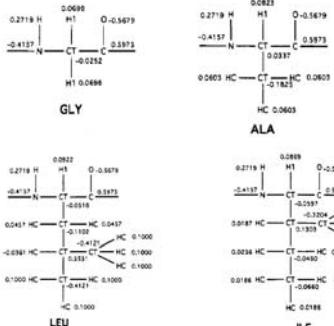


Van der Waals (LJ) Parameters

Van der Waals Parameters							
atom type	R^a	r^b	atom type	R^a	r^b	atom type	R^a
CA	1.9080	0.0840	H1	1.2875	0.0157	HS	0.4000
OB	0.9800	0.0860	H2	1.1830	0.0157	HS	0.4000
CM	1.9080	0.0860	H3	1.4290	0.0150	IP	1.8860
CD	1.9080	0.0860	H4	1.4290	0.0150	IP	1.8860
CT	1.9080	0.0860	H5	1.4590	0.0150	IP	1.1370
F	1.75	0.061	H6	1.4870	0.0157	NS	1.8240
HN	0.0000	0.0000	H7	1.4870	0.0157	NS	1.8240
HH	1.2879	0.0157	HS	1.0000	0.0157	Q	1.6612
						SH	2.0000

$$V_{LJ} = \sum_{i < j} 4\epsilon_{ij} \left[\frac{\sigma_{ij}^{12}}{r_{ij}^{12}} - \frac{\sigma_{ij}^6}{r_{ij}^6} \right]$$

Atomic Partial Charges



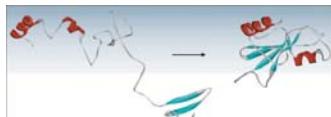
Typical Time Scales

- Bond stretching: $10^{-14} - 10^{-13}$ sec.
- Elastic vibrations: $10^{-12} - 10^{-11}$ sec.
- Rotations of surface sidechains: $10^{-11} - 10^{-10}$ sec.
- Hinge bending: $10^{-11} - 10^{-7}$ sec.
- Rotation of buried side chains: $10^{-4} - 1$ sec.
- Protein folding: $10^{-6} - 10^2$ sec.

Timescale in MD:

- A Typical timestep in MD is $1 \text{ fs (} 10^{-15} \text{ sec)}$
(ideally 1/10 of the highest frequency vibration)

Ab initio protein folding simulation



Physical time for simulation	10^{-4} seconds
Typical time-step size	10^{-15} seconds
Number of MD time steps	10^{11}
Atoms in a typical protein and water simulation	32,000
Approximate number of interactions in force calculation	10^9
Machine instructions per force calculation	1000
Total number of machine instructions	10^{23}
BlueGene capacity (floating point operations per second)	1 petaflop (10^{15})

