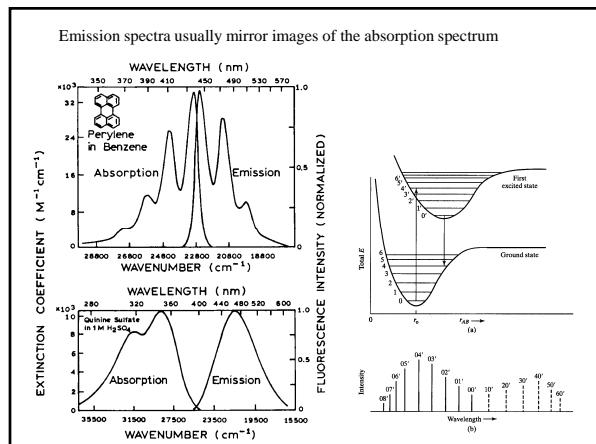
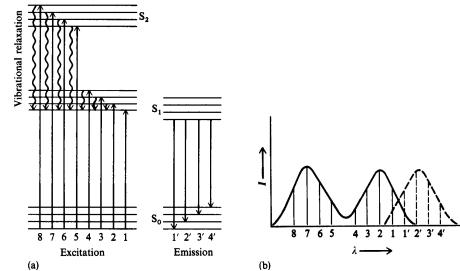
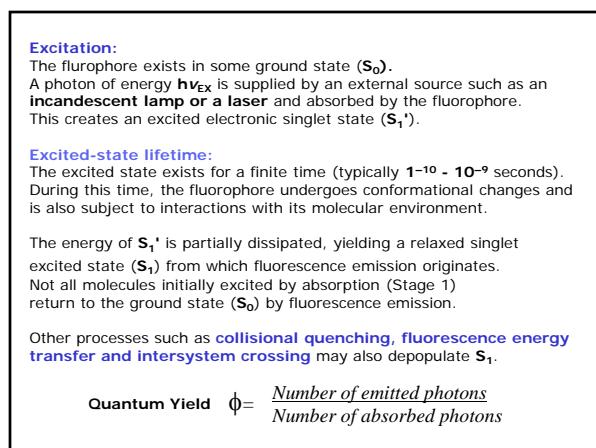
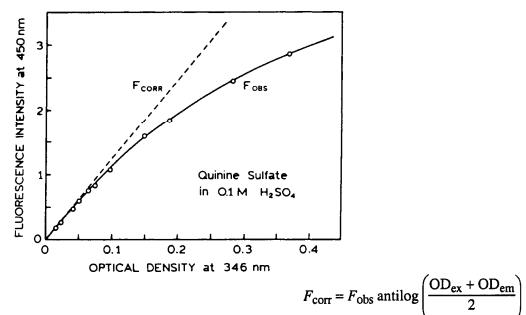


The emission spectrum is always displaced to lower energy.

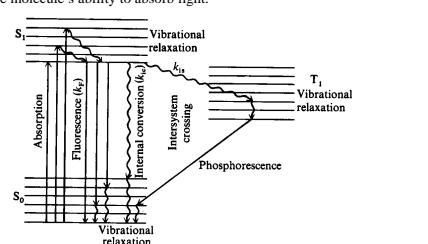


The inner filter effect  
 Fluorescence studies prone to self-absorption artifacts



**Properties of a strongly fluorescing molecule**  
 Large molar absorptivity (extinction coefficient,  $\epsilon$ ). The excited state must be populated for fluorescence to occur.

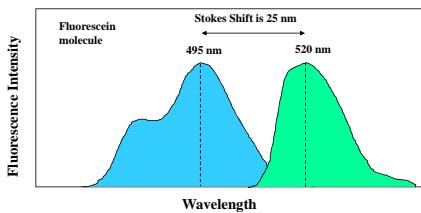
Einstein coefficients - probability of spontaneous emission of light from the excited state of a molecule is proportional to the molecule's ability to absorb light.



### Stokes Shift

- is the energy difference between the lowest energy peak of absorbance and the highest energy of emission

The Stokes shift is fundamental to the sensitivity of fluorescence techniques because it allows **emission photons** to be detected isolated from excitation photons.



### Quantum Yield

From Einstein's coefficients

$$A_{ba} = 8\pi\nu^3 c^{-1} B_{ab}$$

$$A_{ba} = k_F \equiv \text{fluorescence rate constant} = 1/\tau_F$$

where  $\tau_F$  is the "radiative lifetime"

The radiative lifetime is the lifetime that would be observed if all of the molecules returned to the ground state via fluorescence.

The actual, or observed, lifetime is symbolized  $\tau$ :

$$\tau = \frac{1}{k} = \frac{1}{k_F + \sum_i k_i}$$

Having absorbed a photon, what is the likelihood of a photon being re-emitted?

This probability is given by the "quantum yield" ( $\phi_F$ ) for the system:

$$\phi_F = \frac{k_F}{k_F + k_{ic} + k_{is} + k_q[Q]} = \frac{\tau}{\tau_F}$$

### Biological fluorophores

#### Intrinsic fluorophores

##### Proteins

Tryptophan dominates protein fluorescence spectra

- high molar absorptivity
- moderate quantum yield
- ability to quench tyrosine and phenylalanine emission by energy transfer.

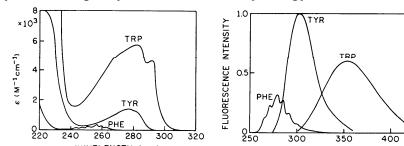


Table 3.1. Fluorescence Parameters of Aromatic Amino Acids in Water at Neutral pH<sup>a</sup>

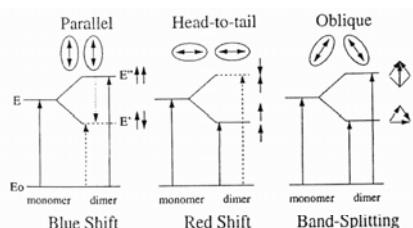
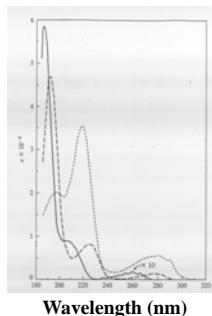
Species	$\lambda_{\text{ex}}$ (nm)	$\lambda_{\text{em}}$ (nm)	Bandwidth (nm)	Quantum yield	Lifetime (ns)
Phenylalanine	269	282	—	0.02	6.8
Tyrosine	275	304	34	0.14	3.6
Tryptophan	295	353	60	0.13	3.1 <sup>b</sup>

### Aromatic amino acids

The amino acids phenylalanine, tyrosine, and tryptophan have  $\pi-\pi^*$  transitions.

There is a pattern of weak bands from 240 - 300 nm and much more intense bands between 190 - 220 nm.

The weak bands are allowed by vibronic coupling (L).



Two dipoles brought **side by side**, relative orientation (parallel vs. antiparallel) - the energy of individual molecules,  $E$ , will split into two new states with the energies  $E''$  and  $E'$ . parallel dipoles repel make up an overall higher dipole moment *i.e.* stronger absorption - higher energy state - **blue shift of absorption** antiparallel dipoles attract cancel each other to make a weak absorption - lowers the energy of that state - **red shift of absorption**

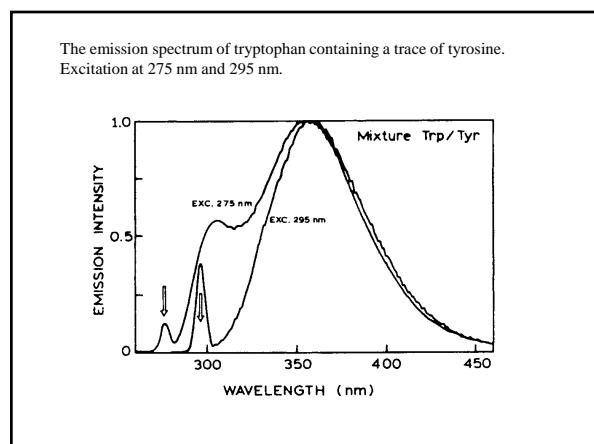
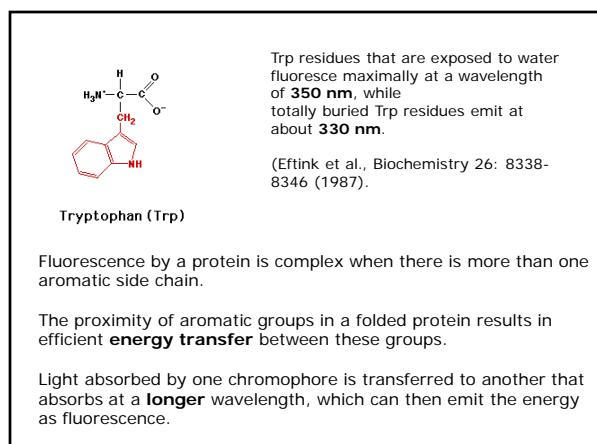
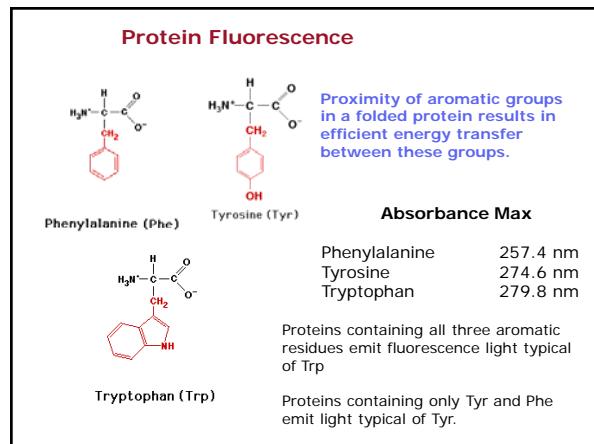
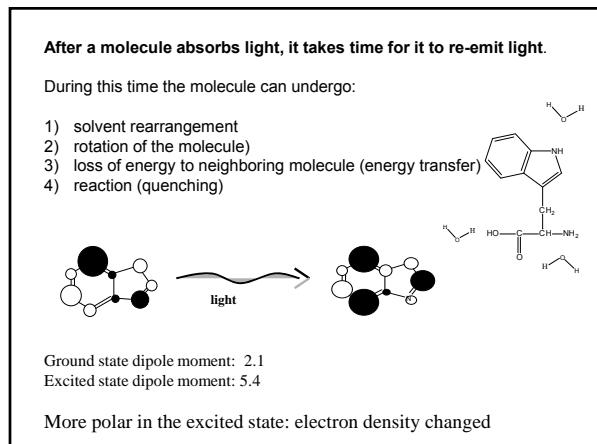
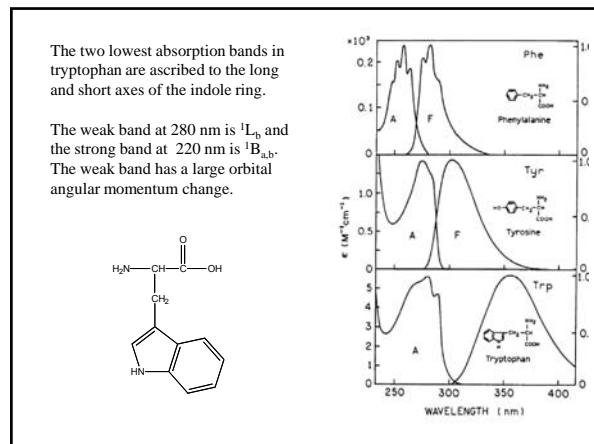
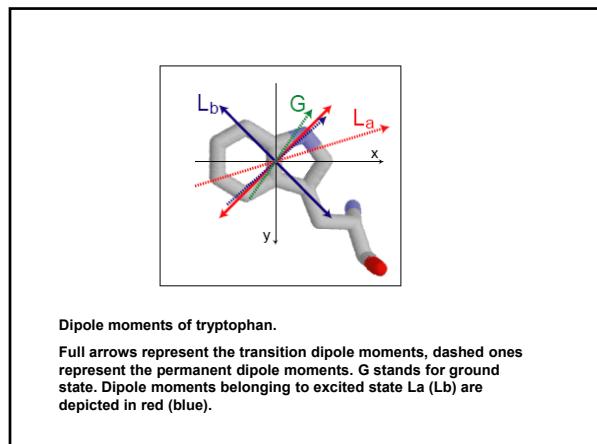
### Tryptophan:

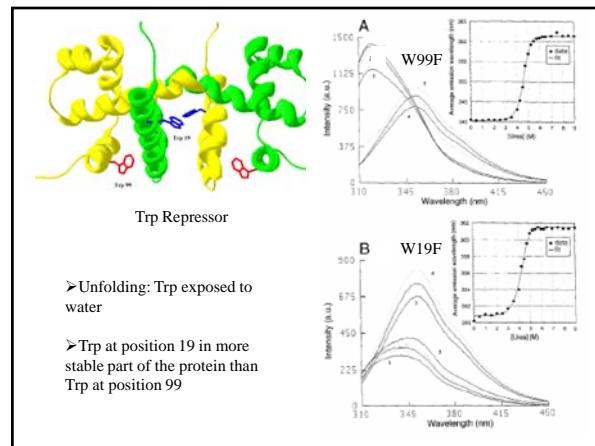
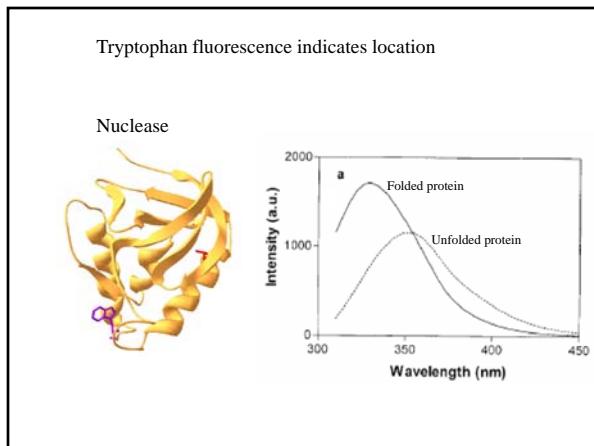
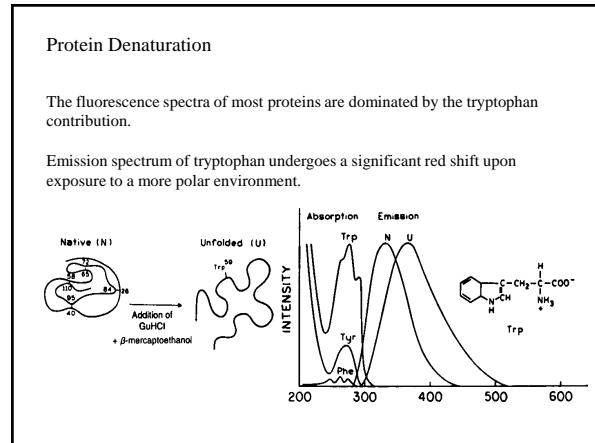
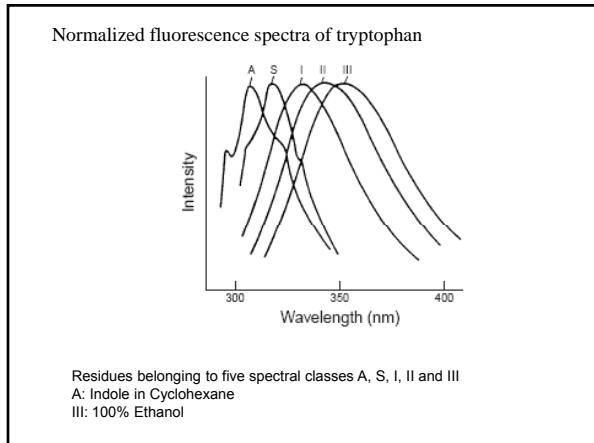
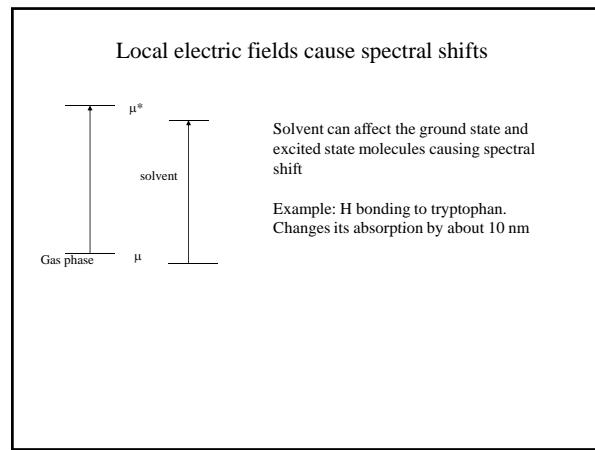
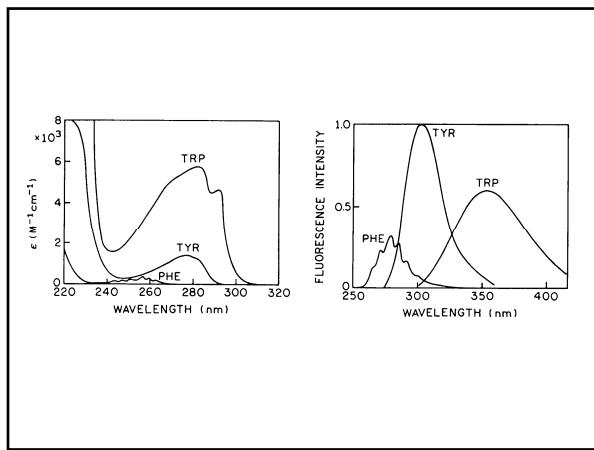
Described as a three level system: the ground state, and two excited states  $L_a$ , and  $L_b$ , with permanent and transition dipole moments given in the Table. The orientations of the dipole moments in the molecular frame are different.

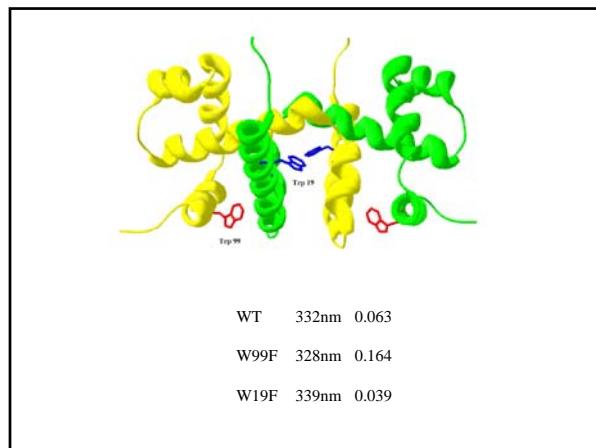
Since the  $L_a$  and  $L_b$  levels are nearly degenerate, their transition energies are taken to be both at 280 nm

state/transition	notation	direction	magnitude (D)
$ 0\rangle$	$\vec{\mu}_0$	(0.690,-0.723,0.0)	2.0
$L_a$	$\vec{\mu}_a$	(0.956,-0.294,0.0)	3.5
$L_b$	$\vec{\mu}_b$	(0.751,-0.660,0.0)	3.5
$ 0\rangle \rightarrow L_a$	$\vec{\mu}_{0a}$	(0.707,-0.707,0.0)	5.5
$ 0\rangle \rightarrow L_b$	$\vec{\mu}_{0b}$	(0.707,0.707,0.0)	2.5

Unit vectors in the direction of the different dipole moments for the lowest transition in tryptophan.

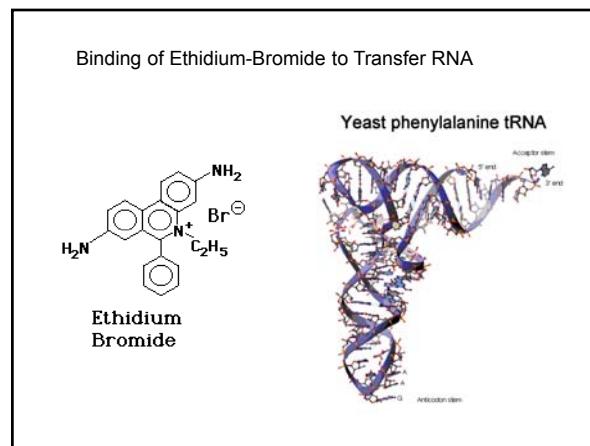
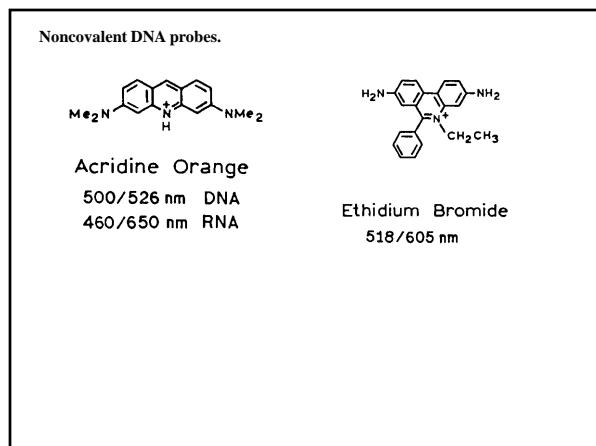
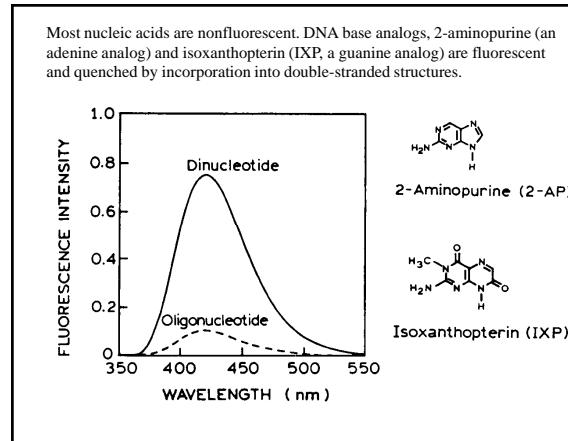
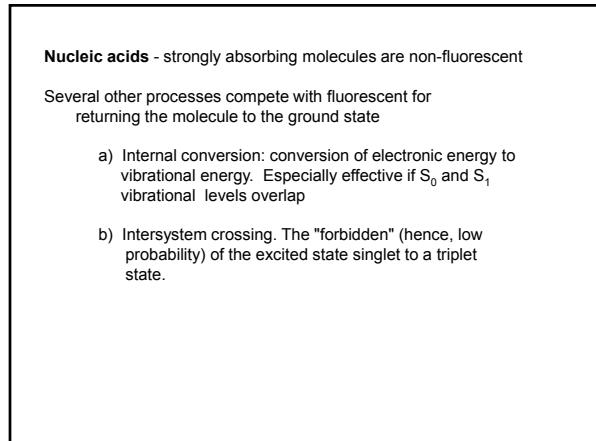
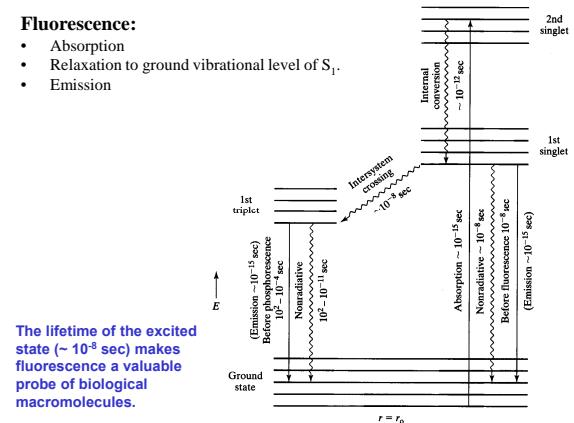


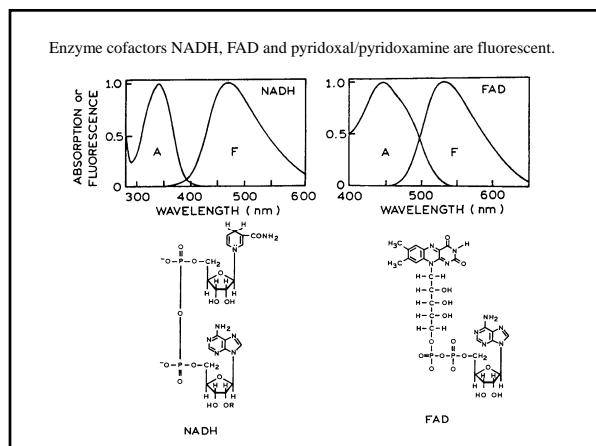
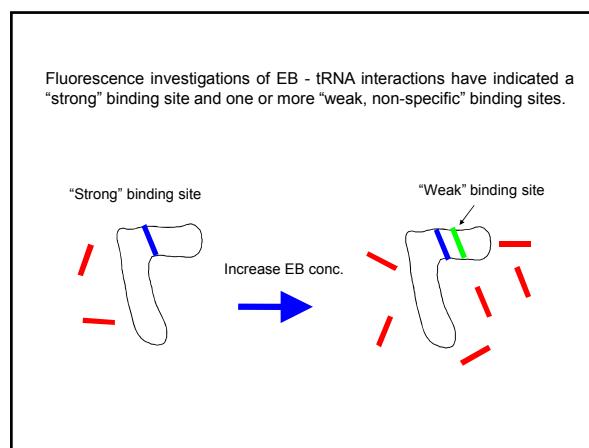
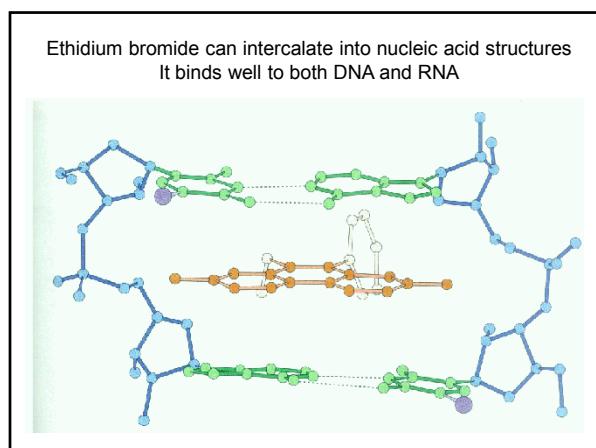




### Fluorescence:

- Absorption
- Relaxation to ground vibrational level of  $S_1$ .
- Emission





**Extrinsic probes: Protein labeling reagents**

**Table 5.2 Typical fluorescent probes<sup>a</sup>**

Probe	Uses	Absorption $\lambda_{\text{max}}$ ( $\times 10^{-3}$ )	Emission <sup>b</sup> $\lambda_{\text{max}}$ (nm)	$\phi_f$	$\tau_f$ (ns)	Sensitivity $\epsilon_{\text{max}} \phi_f$ ( $\times 10^{-3}$ )
Dansyl chloride	Covalent attachment to protein: Lys, Cys	330	3.4	510	0.1	13
1,5-I-AEDANS	—	360	6.8	480	0.5	15
7-Chloro-1,3-diazolo-2-oxa-1,3-diazole (NBD)	Lys, Tyr	345	9.5	—	—	34
Fluorescein isothiocyanate (FITC)	Covalent attachment to protein: Lys	495	42	516	0.3	4
8-Amino-1-naphthalene sulfonate (ANS)	Noncovalent binding to proteins	374	6.8	454	0.98	16
Pyrene and various derivatives	Polarization studies in membranes	342	40	383	0.25	100
Ethenoadenosine and various derivatives	Analogs of nucleotides bind to proteins. Incorporate into nucleic acids	300	2.6	410	0.40	26
Ethidium bromide	Noncovalent binding to nucleic acids	515	3.8	600	~1	26.5
Proflavine monosemicarbazide	Covalent attachment to RNA 3'-ends	445	15	516	0.02	—

**Fluorogenic probes**

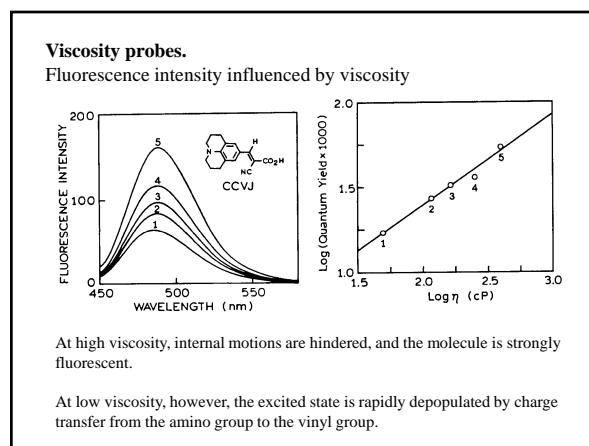
Non-fluorescent compounds that become fluorescent, or release fluorescent species, upon chemical modification.

**Table 3.3. Fluorogenic Amino Reagents<sup>a</sup>**

Compound <sup>b</sup>	$\lambda_{\text{ex}} \text{ max}$ (nm)	$\epsilon$ ( $M^{-1} \text{ cm}^{-1}$ )	$\lambda_{\text{em}} \text{ max}$ (nm)
Fluorescamine	395	6,300	480
MDPF	290, 390	6,400	480
CBQCA	465	—	560
NBD	470	12,900	550

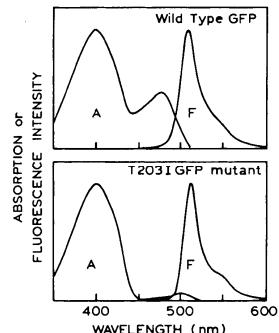
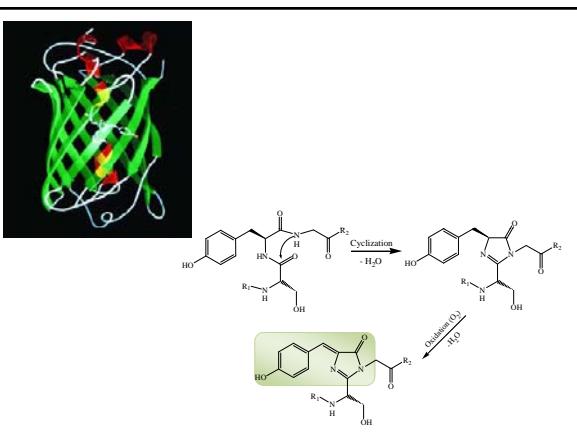
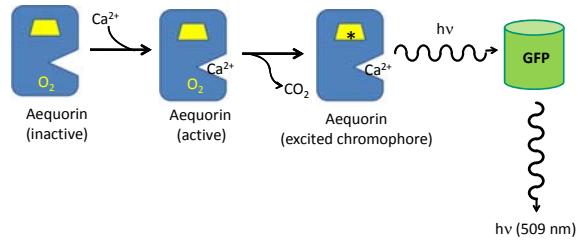
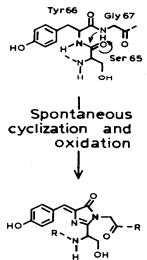
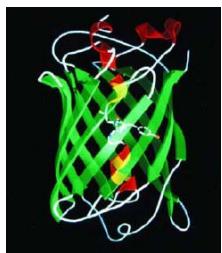
<sup>a</sup>From Ref. 21, p. 127 and Ref. 20 p. 42.  
<sup>b</sup>MDPF, 2-methoxy-2,4-diphenyl-3-(2H)-furanone; CBQCA, 3-(4-carboxybutyoyl)quinoline-2-carboxaldehyde; NBD, 7-Nitrobenz-2-oxa-1,3-diazol-4-yl.

Fluorogenic amino reagents provide sensitive detection of amino acids, peptides, and proteins.



**Green Fluorescent Protein (GFP)** -bioluminescent jellyfish *Aequorea victoria*.

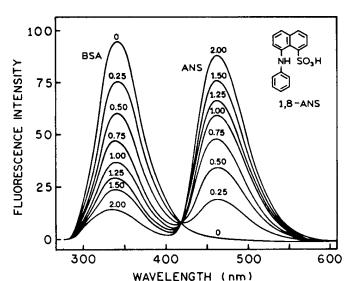
- Obvious  $\beta$ -barrel structure, with chromophore housed within the barrel.
- Remarkably, the chromophore is formed spontaneously (from Ser-65, Tyr-66, Gly-67) upon folding of the polypeptide chain, without the need for enzymatic synthesis.
- It is possible to insert the gene for GFP into cells and use the resulting protein as a reporter for a variety of applications.



Binding of ANS (anilinonaphthalene sulfonic acid) by bovine serum albumin (BSA)

$\lambda_{ex}=280$  nm

Emission from ANS alone in aqueous solution is very weak. In the presence of BSA, the fluorescence of ANS is increased, and the wavelength maximum is shifted to lower wavelength ("blue shift"), the result of adsorption onto nonpolar regions of the BSA surface.



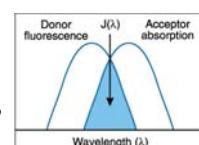
Tryptophan emission of serum albumin decreases with the binding of ANS. Example of **resonant energy transfer**.

### Fluorescence Resonance Energy Transfer (FRET)

#### Primary Conditions for FRET

Donor and acceptor molecules must be in close proximity (typically 10–100 Å).

The absorption spectrum of the acceptor must overlap fluorescence emission spectrum of the donor.



Donor and acceptor transition dipole orientations must be approximately parallel.

#### Förster Radius

The distance at which energy transfer is 50% efficient (i.e., 50% of excited donors are deactivated by FRET) is defined by the Förster radius ( $R_0$ ).

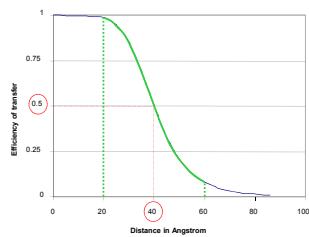
The magnitude of  $R_0$  is dependent on the spectral properties of the donor and acceptor dyes.

### The distance dependence of the energy transfer efficiency ( $E$ )

$$r = \left( \frac{1}{E} - 1 \right)^{1/6} R_0$$

Where  $r$  is the distance separating the centers of the donor and acceptor fluorophores,  $R_0$  is the Förster distance.

The efficiency of transfer varies with the inverse sixth power of the distance.



$R_0$  in this example was set to 40 Å.

When the  $E$  is 50%,  $R=R_0$

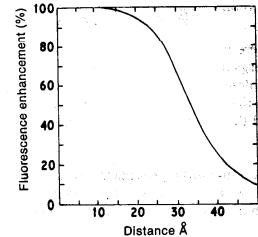
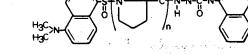
Distances can generally be measured between  $\sim 0.5 R_0$  and  $\sim 1.5 R_0$

PM

### Distance-dependence

was demonstrated by Lubert Stryer

Energy is transferred between the naphthyl group (right) and the dansyl group



The proline spacers adopt a helical configuration, permitting calculation of the average distance between the donor and acceptor groups for a given length spacer.

The measured efficiencies (in this case the enhancement of the dansyl fluorescence) confirmed the expected  $1/r^6$  dependence.

### Fluorescence Resonant Energy Transfer (FRET, RET)

FRET is a process by which excitation energy is transferred from one chromophore (the donor chromophore) to another chromophore (the acceptor chromophore).

The donor must be fluorescent; the acceptor need not be.

If the emission spectrum of the donor overlaps the absorption spectrum of the acceptor, energy can be transferred from the donor to the acceptor

- 1) The energy transfer process is non-radiative dipole-dipole interaction.
- 2) The probability of energy transfer depends on the extent of overlap of the donor emission and acceptor absorption spectrum.
- 3) Energy transfer has two manifestations:
  - The intensity of the donor emission spectrum is reduced.
  - Fluorescence emission by the acceptor - called "sensitized emission."
- 4) Energy does not go back and forth from donor to acceptor.
- 5) The efficiency of energy transfer is strongly dependent on the distance between the donor and acceptor and the orientation of the donor and acceptor transition dipoles.  $E \propto 1/r^6$

### Theory

In the absence of energy transfer, the lifetime is  $\tau$ .

In the presence of acceptor,  $\frac{1}{\tau_T} = \frac{1}{\tau} + k_T$ , and  $k_T = \frac{1}{\tau_T} - \frac{1}{\tau}$

The efficiency ( $E_T$ ) of depopulation by RET is then

$$E_T = \frac{k_T}{1/\tau + k_T} = \frac{1/\tau_T - 1/\tau}{1/\tau_T}$$

$$\text{So } \frac{\tau_T}{\tau} = 1 - E_T$$

$$\text{Since } \phi_D = \frac{1}{\tau} \text{ and } \phi_T = \frac{1}{\tau_T}, \text{ then } \frac{\phi_T}{\phi_D} = \frac{I_T}{I_D} = 1 - E_T$$

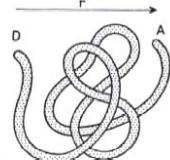
Define  $R_o$  as the distance at which energy transfer is 50% efficient.

$$\text{At } R_o, k_T = \frac{1}{\tau}$$

$$\text{also, } k_T \propto \left( \frac{1}{r^6} \right), \therefore k_T = \frac{C}{R_o^6}$$

$$\therefore \frac{1}{\tau} = \frac{C}{R_o^6} \text{ so that } C = \frac{1}{\tau} R_o^6$$

$$\text{Thus, } k_T = \frac{1}{\tau} \left( \frac{R_o^6}{r^6} \right)$$



Substituting back into the earlier expression for  $E$ :

$$E = \frac{k_T}{\tau^{-1} + k_T} = \frac{R_o^6}{R_o^6 + r^6}$$

$$r = R_o \left( \frac{1 - E^{1/6}}{E} \right)^{1/6}$$

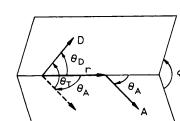
The magnitude of  $R_o$  depends on the degree of overlap between the donor emission spectrum and the acceptor absorption spectrum, which is given by the overlap integral,  $J$ . For calculating  $J$ , the normalized emission spectrum is used

$$J = \int_0^{\infty} F_{D,norm}(\lambda) \epsilon_A(\lambda) \lambda^4 d\lambda = \frac{0}{\int_0^{\infty} F_D(\lambda) d\lambda}$$

$$R_o^6 = \frac{9000(\ln 10)\kappa^2 Q_D}{128\pi^5 N n^4} \int_0^{\infty} F_{D,norm}(\lambda) \epsilon_A(\lambda) \lambda^4 d\lambda$$

$$R_o = 9.78 \times 10^3 [\kappa^2 n^{-4} Q_D J]^{1/6} \text{ (in Å)}$$

$$R_o^6 = 8.79 \times 10^{23} [\kappa^2 n^{-4} Q_D J] \text{ (in Å}^6\text{)}$$



$n$  is the refractive index, and  $Q_D$  is the donor quantum yield in the absence of acceptor.

$\kappa^2$  is the "orientation factor". Depending on the geometry of the donor/acceptor transition dipoles,  $\kappa^2$  can assume values from 0 to 4. It is usually assumed that  $\kappa^2 = 2/3$ , the value for donors and acceptors that randomize their orientations prior to energy transfer.

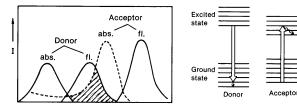
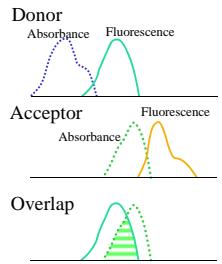
The overlap integral  $J$  is defined by:

$$J = \int_0^{\infty} f_D(\lambda) \epsilon_A(\lambda) \lambda^4 d\lambda$$

Where  $\lambda$  is the wavelength of the light,  $\epsilon_A(\lambda)$  is the molar extinction coefficient at that wavelength and  $f_D(\lambda)$  is the fluorescence spectrum of the donor normalized on the wavelength scale:

$$f_D(\lambda) = \frac{F_{D\lambda}(\lambda)}{\int_0^{\infty} F_{D\lambda}(\lambda) d\lambda}$$

Where  $F_{D\lambda}(\lambda)$  is the donor fluorescence per unit wavelength interval



## Applications

-Estimation of inter-chromophore distances in biological macromolecules, using intrinsic and extrinsic probes

-Strategy for assaying molecular proximity

-Biological phenomena that have been examined by FRET include:

- protein-ligand interactions
- protein-protein interactions
- protein folding/unfolding
- DNA denaturation/renaturation
- protein-membrane interactions
- membrane diffusion (both protein and lipid)

- Diffusion-enhanced energy transfer from long-lived donors (e.g., lanthanides) allows the distance of a chromophore from the surface of a macromolecule to be determined.

## Quenching:

Certain moieties are particularly efficient at de-exciting the excited state - mostly through interaction with the electron in the excited state orbital.

High-molecular weight ions or triplet species can be effective quenchers. Examples include  $O_2$ , I, and acrylamide.

## Quenching

Two general types: Dynamic and Static

$$\frac{F_o}{F} = 1 + k_q \tau_o [Q] = 1 + K_D [Q]$$

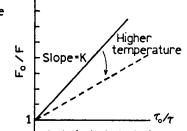
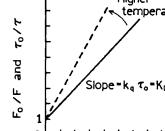
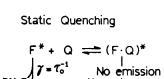
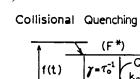
**Dynamic** (collisional, Stern-Volmer)

Measures accessibility of fluorophore.

**Static** (complex formation)

$$\frac{F_o}{F} = 1 + K_s [Q]$$

$$K_s = \frac{[F \cdot Q]}{[F][Q]}$$

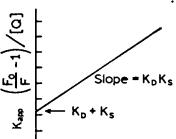
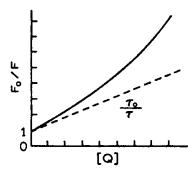
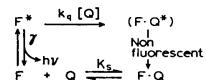


## Combined dynamic and static quenching

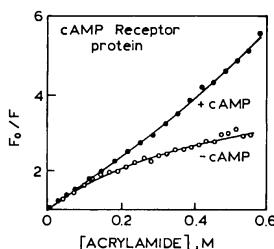
$$\frac{F_o}{F} = (1 + K_D [Q])(1 + K_s [Q])$$

$$= 1 + (K_D + K_s)[Q] + K_D K_s [Q]^2$$

$$= 1 + K_{app} [Q], \text{ where } K_{app} = \left( \frac{F_o}{F} - 1 \right) \frac{1}{[Q]}$$

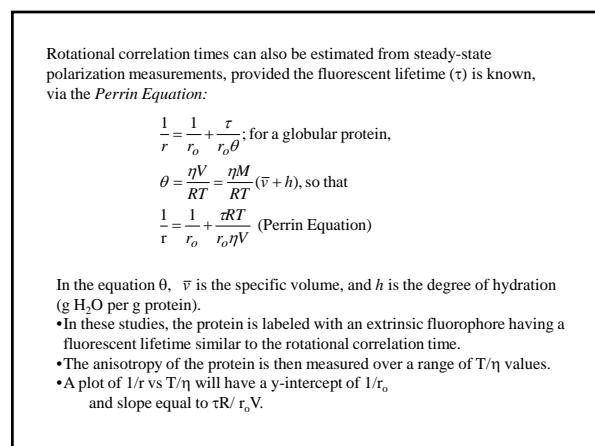
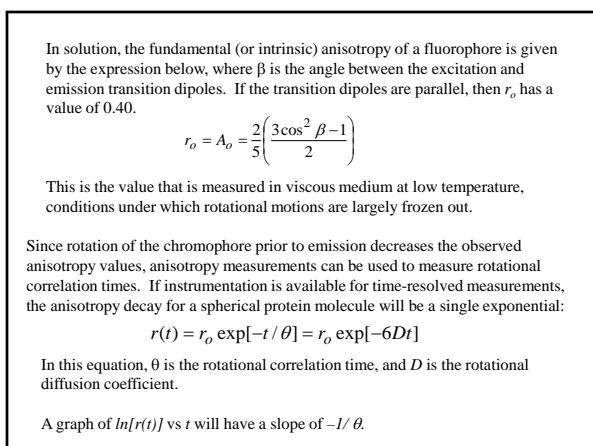
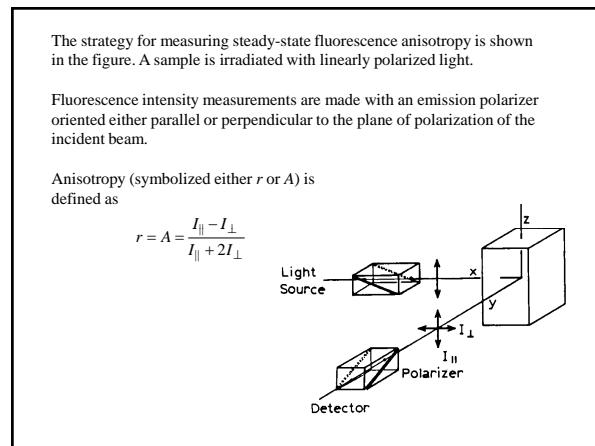
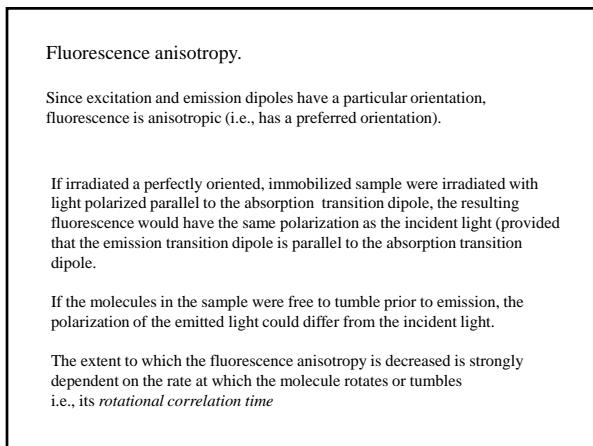
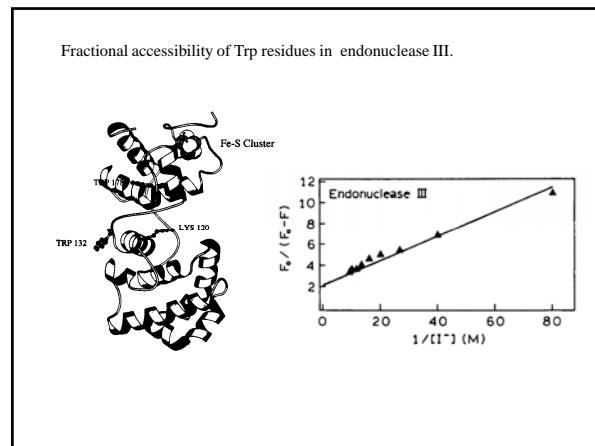
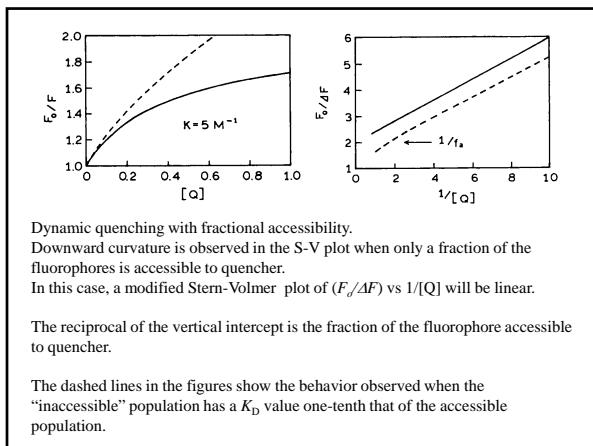


Quenching can also be used to recognize conformational changes that occur with substrate, ligand, or effector binding.



Data for the cAMP receptor protein with two Trp

In the absence of cAMP, only one of the two Trp residues are quenched by acrylamide. Addition of cAMP causes both chromophores to become sensitive to the quencher.



The method can be used even if the fluorophore is depolarized by segmental (i.e., local) motion, as well as by the overall tumbling of the protein molecule, provided that the segmental motions are much faster than the rotational diffusion.

Table 10.4. Calculated Rotational Correlation Times for Proteins

T	Molecular weight (kDa)	Correlation time $\theta^*$ (ns)		
		$\eta = 0$	$\eta = 0.2$	$\eta = 0.4$
2 °C	10	5.5	6.9	8.4
	25	13.7	17.3	21.4
	50	27.4	34.6	42.0
	100	54.8	69.2	84.0
	500	274.0	346.0	420.0
20 °C	10	3.1	3.9	4.7
	25	7.0	9.7	11.8
	50	15.4	19.5	23.6
	100	30.8	39.0	47.2
	500	154.0	195.0	234.0
37 °C	10	2.0	2.5	3.1
	25	5.0	6.4	7.7
	50	10.0	12.7	15.4
	100	20.1	25.4	30.4
	500	100.5	127.0	154.0

<sup>a</sup>Calculated using  $\theta = n(M + \eta)/87$  with  $\eta = 0.73$  cP and the indicated degree of hydration ( $\eta$ ). The viscosities are  $\eta(20 °C) = 1.00$  cP,  $\eta(20 °C) = 1.00$  cP, and  $\eta(37 °C) = 0.69$  cP.

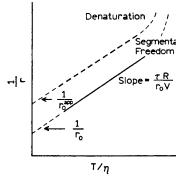


Table 10.5. Rotational Correlation Times for Proteins<sup>a</sup>

Protein	Molecular weight (m)	Observed $\theta$ (ns)	$\theta_{obs}/\theta_{cal}$
Apomyoglobin	17,000	8.3	1.9
$\beta$ -Lactoglobulin (monomer)	18,400	8.5	1.8
Trypsin	25,000	12.9	2.0
Chymotrypsin	25,000	15.1	2.3
Carbonic anhydrase	30,000	11.2	1.4
$\beta$ -Lactoglobulin (dimer)	36,000	20.8	2.1
Apperoxidase	40,000	25.2	2.4
Serum albumin	66,000	41.7	2.4

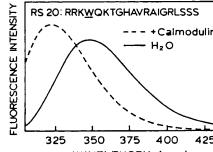
<sup>a</sup>From Ref. 30.

<sup>b</sup> $\theta_{obs}$  is the observed rotational correlation time, adjusted to the value of  $\theta_{cal}$  corresponding to water at 25 °C.  $\theta_{cal}$  is the rotational correlation time calculated for a rigid, unhydrated sphere with the molecular weight of the protein, assuming a partial specific volume of 0.73 mL/g.

#### Examples:

The figure shows the blue-shift that occurs when the peptide is bound by calmodulin.

The emission maximum shift and the increase in the maximal fluorescence intensity suggest that the tryptophan residue is in a more apolar environment.



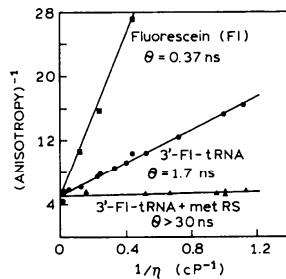
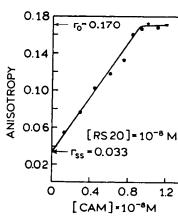
The figure to the right shows the changes in the anisotropy of a peptide fragment of myosin light-chain kinase upon titration with calmodulin, which binds the peptide.

The increase in the anisotropy indicates that the peptide interacts tightly with calmodulin.

The maximal anisotropy appears at a calmodulin:peptide ratio of 0.93, implying stoichiometry not greater than 1:1.

The shape of the curve suggests that at a calmodulin concentration of  $10^{-8}$  M (equivalent to the peptide concentration) there is no free peptide.

The anisotropy of tryptophan emission of the free peptide is 0.033 and of the fully bound peptide 0.170.



The figure above shows the Perrin Plot for fluorescein, Fl-labeled-tRNA, and the Fl-tRNA bound to methionyl-tRNA synthetase.

#### Time-resolved fluorescence spectroscopy

Time-resolved data contain more information than is available from steady-state measurements, e.g., consider a protein containing two Trp residues.

- Because of spectral overlap, it is usually not possible to resolve the signals from the two residues.

- If the two Trp residues have different fluorescent lifetimes, their contributions to the overall fluorescence behavior can be determined from time-resolved measurements.

- One can then determine how each is influenced by the interactions of the protein with substrates, effectors, or other macromolecules.

Since the fluorescence lifetime of a molecule is very sensitive to its molecular environment, measurement of the fluorescence lifetime(s) reveals much about the state of the fluorophore.

Many macromolecular events, such as rotational diffusion, resonance-energy transfer, and dynamic quenching, occur on the same time scale as the fluorescence decay.

Thus, time-resolved fluorescence spectroscopy can be used to investigate these processes and gain insight into the chemical surroundings of the fluorophore.

Fluorescence lifetimes are generally on the order of 1-10 nsec, although they can range from hundreds of nanoseconds to the sub-nanosecond time scale.

#### What is the "lifetime" of a fluorophore?

Absorption and emission processes are almost always studied on *populations* of molecules and the properties of the typical members of the population are obtained from the macroscopic properties of the process.

The behavior of an excited population of fluorophores is described by a rate equation:

$$\frac{dn^*}{dt} = -n^* \Gamma + f(t)$$

where  $n^*$  is the number of excited elements at time  $t$ ,  $\Gamma$  is the rate constant of emission and  $f(t)$  is an arbitrary function of the time, describing the time course of the excitation. The dimensions of  $\Gamma$  are  $\text{sec}^{-1}$  (transitions per molecule per unit time).

If excitation occurs at  $t = 0$ , the equation is:

$$\frac{dn^*}{dt} = -n^* \Gamma$$

and describes the decrease in excited molecules at all further times. Integration gives:

$$n^*(t) = n^*(0) \exp(-\Gamma t)$$

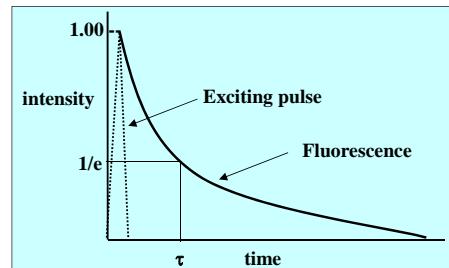
The lifetime,  $\tau$ , is equal to  $\Gamma^{-1}$

If a population of fluorophores are excited, the lifetime is the time it takes for the number of excited molecules to decay to  $1/e$  or 36.8% of the original population according to:

$$\frac{n^*(t)}{n^*(0)} = e^{-t/\tau}$$

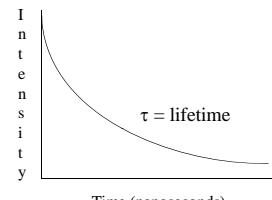
In pictorial form:

$$\frac{n^*(t)}{n^*(0)} = e^{-t/\tau}$$



### Lifetime

- Fluorescence lifetime is the average time that an electron spends in the excited state before a photon is emitted
- Measurement of the fluorescence from a large number of molecules, following a short pulse excitation, will show an exponential decay
- The lifetime is given by the  $1/e$  point of the decay



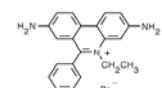
$$\text{Exponential decay: } I = I_0 e^{-t/\tau}$$

The lifetime and quantum yield for a given fluorophore is often dramatically affected by its environment.

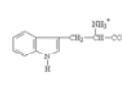
Examples of this fact would be NADH, which in water has a lifetime of  $\sim 0.4$  ns but bound to dehydrogenases can be a long as 9 ns.



ANS in water is  $\sim 100$  picoseconds but can be 8 – 10 ns bound to proteins



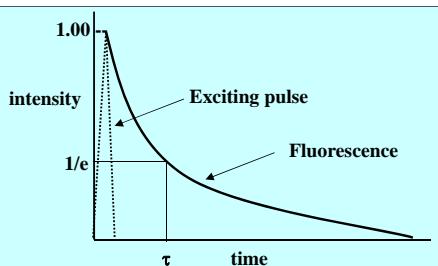
Ethidium bromide is 1.8 ns in water, 22 ns bound to DNA and 27 ns bound to tRNA



The lifetime of tryptophan in proteins ranges from  $\sim 0.1$  ns up to  $\sim 8$  ns

Excited state lifetimes have traditionally been measured using either the *impulse* response or the *harmonic* response method. In principle both methods have the same information content. These methods are also referred to as either the "time domain" method or the "frequency domain" method.

In the *impulse* (or pulse) method, the sample is illuminated with a short pulse of light and the intensity of the emission versus time is recorded. Originally these short light pulses were generated using *flashlamps* which had widths on the order of several nanoseconds. Modern laser sources can now routinely generate pulses with widths on the order of picoseconds or shorter.

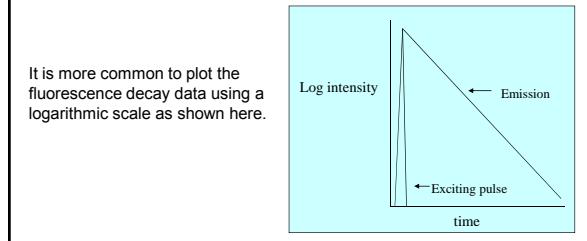


As shown in the intensity decay figure, the *fluorescence* lifetime,  $\tau$ , is the time at which the intensity has decayed to  $1/e$  of the original value. The decay of the intensity with time is given by the relation:

$$I_t = \alpha e^{-t/\tau}$$

Where  $I_t$  is the intensity at time  $t$ ,  $\alpha$  is a normalization term (the pre-exponential factor) and  $\tau$  is the lifetime.

It is more common to plot the fluorescence decay data using a logarithmic scale as shown here.



Quenching phenomena are best studied using time-resolved measurements, because one can readily distinguish static and dynamic quenching.

Formation of static ground-state complexes **does not decrease the decay time** of the uncomplexed fluorophores because only the unquenched fluorophores are observed in a fluorescence experiment.

Dynamic quenching is a rate process acting on the entire excited-state population and thus **decreases the mean decay time** of the excited-state population.

RET is also best studied by time-resolved measurements.

e.g. if a protein contains a donor and acceptor, and the steady-state measurements indicate that the donor is 50% quenched by the acceptor.

- the observation of 50% donor quenching can be due to 100% quenching for one-half of the donors or to 50% quenching of all the donors, or some combination of these two limiting cases.

- Steady-state data cannot distinguish between these two cases. However, very different donor intensity decays would be observed for each case.

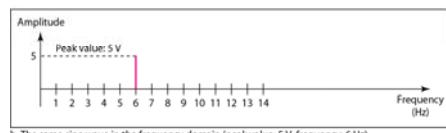
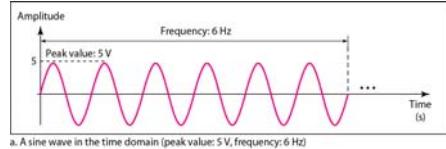
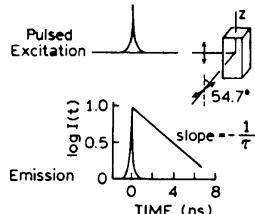
The down side to time-resolved measurements is that the instrumentation is highly specialized and costly.

Two experimental strategies: **time-domain** and **frequency-domain**.

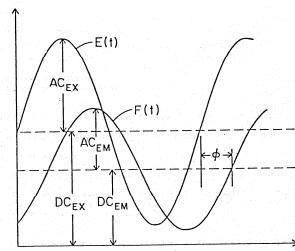
Time-domain measurements are conceptually simpler.

The sample is irradiated with a brief pulse of light, then subsequent (exponential) fluorescent decay is monitored.

The general technique is called "time-correlated single photon counting" (TCSPC)



Fluorophores subjected to such an excitation will give rise to a modulated emission which is shifted in phase relative to the exciting light  
- This is due to the persistence of the excited state



phase delay ( $\phi$ ) between the excitation,  $E(t)$ , and the emission,  $F(t)$ . Also shown are the AC and DC levels associated with the excitation and emission waveforms.

It can be shown that:

$$F(t) = F_0 [1 + M_F \sin(\omega t + \phi)]$$

This relationship signifies that measurement of the phase delay,  $\phi$ , forms the basis of one measurement of the lifetime,  $\tau$ .

$$\tan \phi = \omega \tau$$

The *modulations* of the excitation ( $M_E$ ) and the emission ( $M_F$ ) are given by:

$$M_E = \left( \frac{AC}{DC} \right)_E \quad \text{and} \quad M_F = \left( \frac{AC}{DC} \right)_F$$

The *relative modulation*,  $M$ , of the emission is then:

$$M = \frac{(AC/DC)_F}{(AC/DC)_E}$$

$\tau$  can also be determined from  $M$  according to the relation:  $\tau = \frac{1}{\sqrt{1 + (M\omega)^2}}$

Using the *phase shift* and *relative modulation* one can thus determine a *phase lifetime* ( $\tau_p$ ) and a *modulation lifetime* ( $\tau_M$ ).

If the fluorescence decay is a single exponential, then  $\tau_p$  and  $\tau_M$  will be equal at all modulation frequencies.

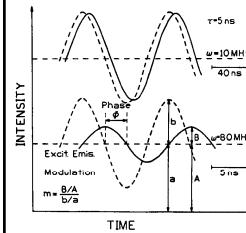
If, however, the fluorescence decay is multiexponential then  $\tau_p < \tau_M$  and, moreover, the values of both  $\tau_p$  and  $\tau_M$  will depend upon the modulation frequency, i.e.,

$$\tau_p(\omega_1) < \tau_p(\omega_2) \quad \text{if } \omega_1 > \omega_2$$

Typical phase and modulation data

Frequency (MHz)	$\tau_p$ (ns)	$\tau_M$ (ns)
5	6.76	10.24
10	6.02	9.70
30	3.17	6.87
70	1.93	4.27

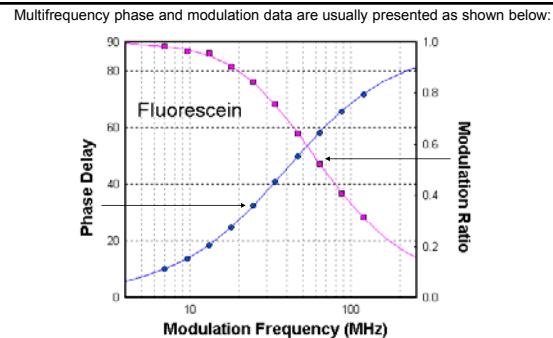
Frequency-domain (FD) measurements. Also called phase modulation.



The sample is excited with intensity-modulated light. The intensity is varied at a high frequency comparable to the reciprocal of the decay time. When a fluorescent sample is excited in this way the emission is forced to respond at the same modulation frequency.

Since the excited state has a finite lifetime, the emission is delayed relative to the excitation. The delay is measured as a *phase shift*,  $\phi$ , which is used to calculate the decay time.

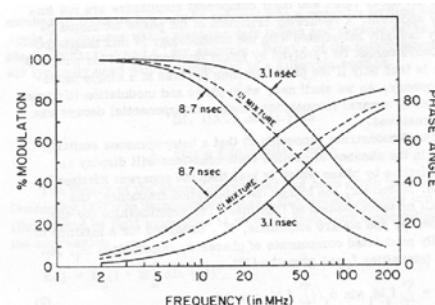
The peak-to-peak height of the emission is also decreased relative to that of the modulated excitation beam. This phenomenon, called *demodulation*,  $m$ , can also be used to calculate the decay time.



The plot shows the frequency response curve (phase and modulation) of Fluorescein in phosphate buffer pH 7.4, using a 470 nm LED. The emission was collected through a 530 high pass filter.

The data is best fitted by a single exponential decay time of 4 ns.

A case of multi-exponential decays is shown here for a system of two lifetime species of 8.7 ns and 3.1 ns and a 1 to 1 mixture (in terms of fractional intensities)



Multifrequency phase and modulation data is usually analyzed using a non-linear least squares method in which the actual phase and modulation ratio data (not the lifetime values) are fit to different models such as single or double exponential decays.

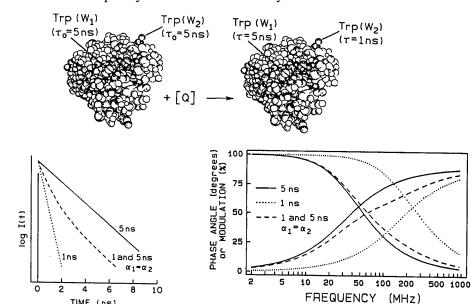
The quality of the fit is then judged by the *chi-square value* ( $\chi^2$ ) which is given by:

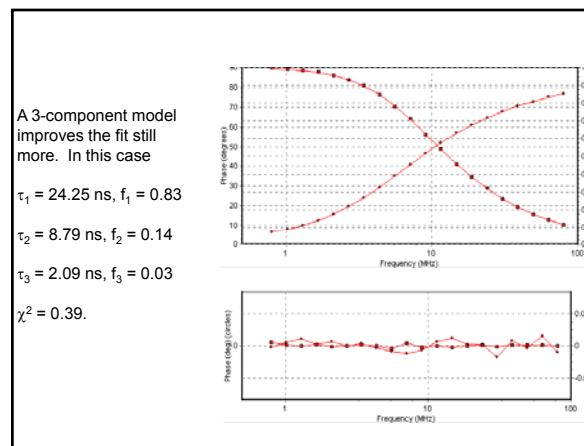
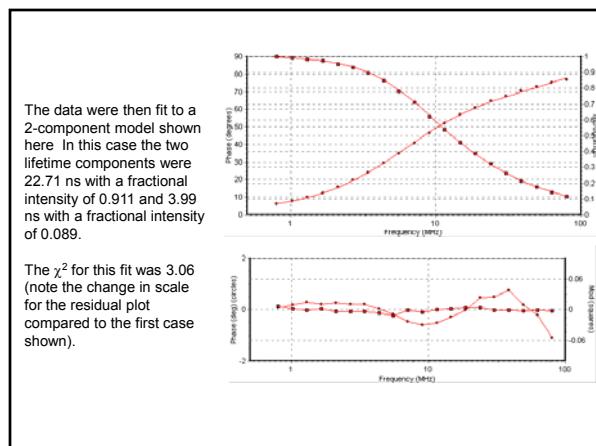
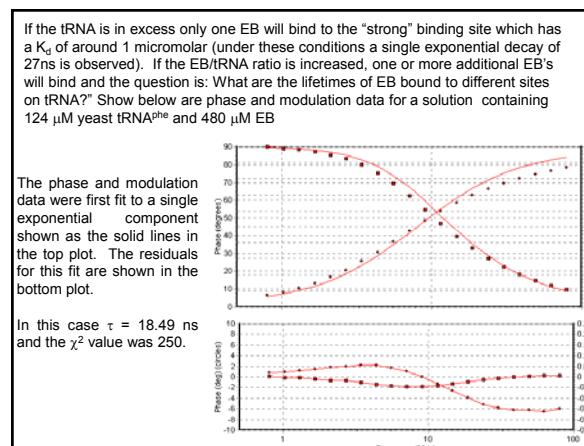
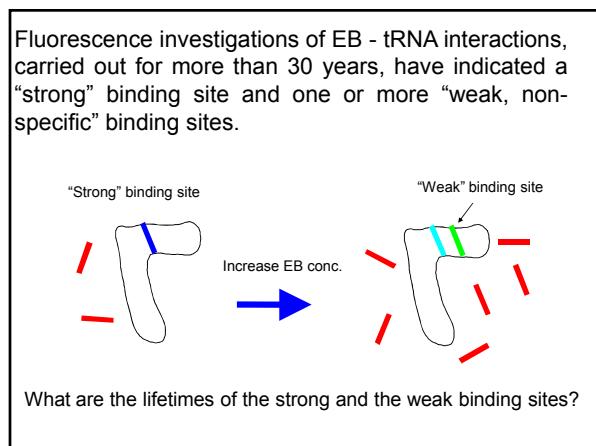
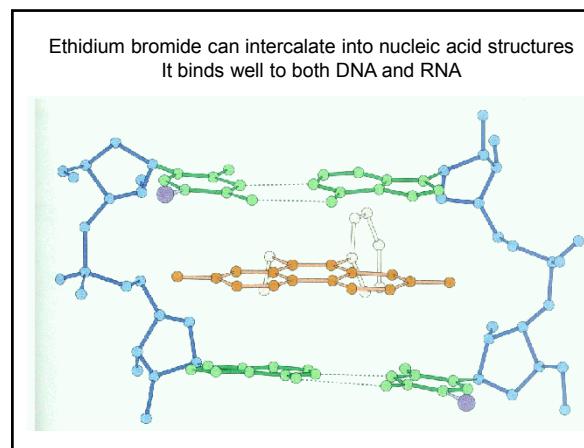
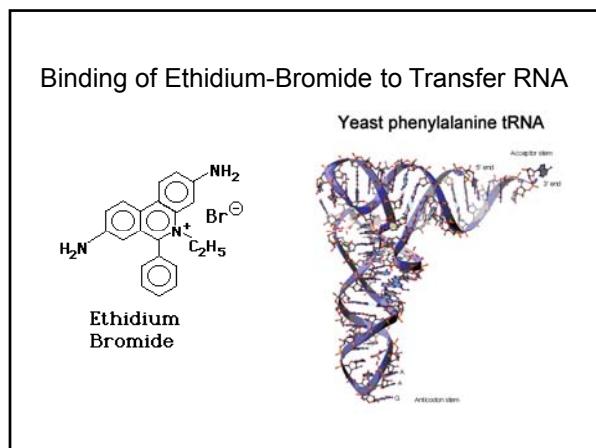
$$\chi^2 = \{[(P_c - P_m)/\sigma^P] + [(M_c - M_m)/\sigma^M]\}/(2n - f - 1)$$

where  $P$  and  $M$  refer to phase and modulation data, respectively,  $c$  and  $m$  refer to calculated and measured values and  $\sigma^P$  and  $\sigma^M$  refer to the standard deviations of each phase and modulation measurement, respectively.  $n$  is the number of modulation frequencies and  $f$  is the number of free parameters.

Consider a protein that contains two Trp residues – one buried, the other accessible. Assume both have 5 nsec lifetimes. Although intrinsically indistinguishable, they could be resolved in the presence of a quencher, which would shorten the excited-state lifetime of the accessible chromophore.

Simulated time- and frequency-domain data for this system are shown below:



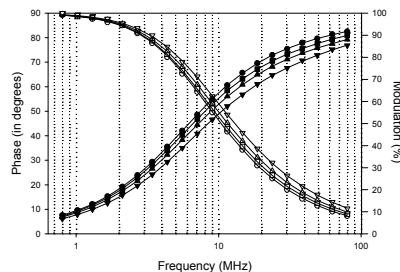


Adding a fourth component – with all parameters free to vary – does not lead to a significant improvement in the  $\chi^2$ . In this case one finds 4 components of 24.80 ns (0.776), 12.13ns (0.163), 4.17 ns (0.53) and 0.88 ns (0.008).

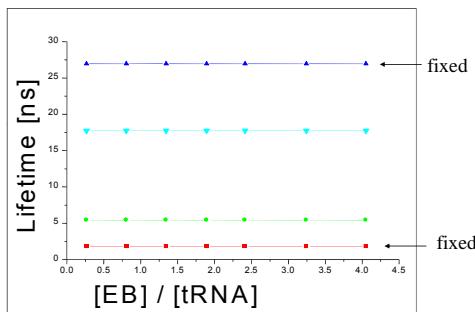
But we are not using all of our information! We can actually fix some of the components in this case. We know that **free EB** has a lifetime of **1.84 ns** and we also know that the lifetime of **EB bound to the “strong” tRNA binding site is 27 ns**. So we can fix these in the analysis. The results are four lifetime components of 27 ns (0.612), 18.33 ns (0.311), 5.85 ns (0.061) and 1.84 ns (0.016). The  $\chi^2$  improves to 0.16.

We can then go one step better and carry out “**Global Analysis**”. In Global Analysis, multiple data sets are analyzed simultaneously and different parameters (such as lifetimes) can be “linked” across the data sets. The important concept in this particular experiment is that the lifetimes of the components stay the same and only their fractional contributions change as more ethidium bromide binds.

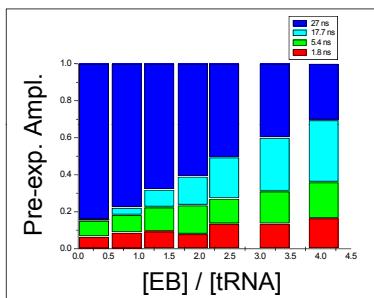
In this system, 8 data sets, with increasing EB/tRNA ratios, were analyzed. Some of the data are shown below for EB/tRNA ratios of 0.27 (circles), 1.34 (squares), 2.41 (triangles) and 4.05 (inverted triangles).



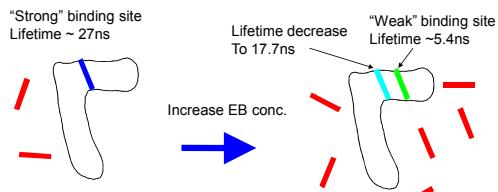
Global Analysis on seven data sets fit best to the 4 component model with two fixed components of 27ns and 1.84ns and two other components of 17.7ns and 5.4ns.



As shown in the plot below, as the EB/tRNA ratio increases the fractional contribution of the 27ns component decreases while the fractional contributions of the 17.7ns and 5.4ns components increase.



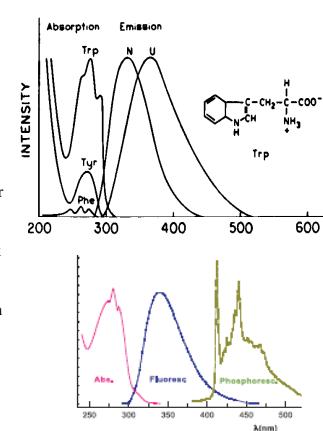
### The Model



Radiative emission can also occur from the excited triplet state, in which case it is called “phosphorescence.”

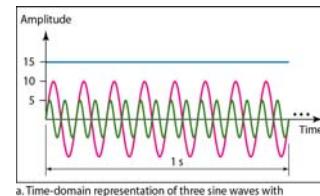
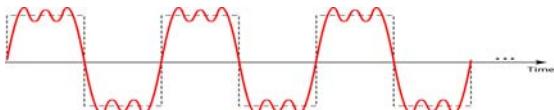
Phosphorescence is characterized by a very long lifetime (seconds or longer), because emission must be accompanied by the unlikely conversion of the triplet state back to a singlet.

It is difficult to observe in solution at room temperature because of internal conversion and “quenching.”

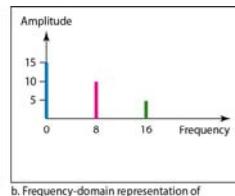


### Fourier Series

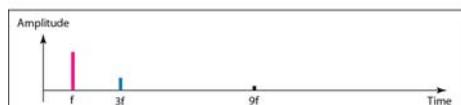
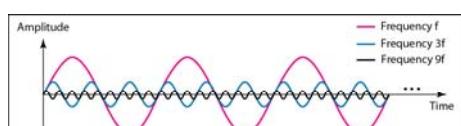
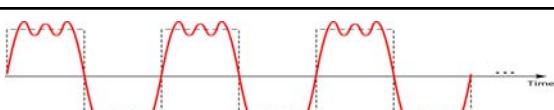
- Every composite periodic signal can be represented with a series of sine and cosine functions.
- The functions are integral harmonics of the fundamental frequency ‘‘f’’ of the composite signal.
- Using the series we can decompose any periodic signal into its harmonics.



a. Time-domain representation of three sine waves with frequencies 0, 8, and 16



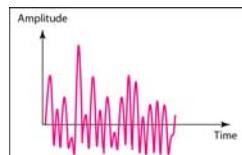
b. Frequency-domain representation of the same three signals



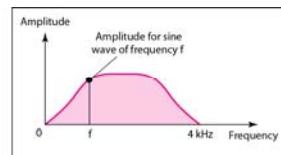
b. Frequency-domain decomposition of the composite signal

### Fourier Transform

- Fourier Transform gives the frequency domain of a nonperiodic time domain signal.

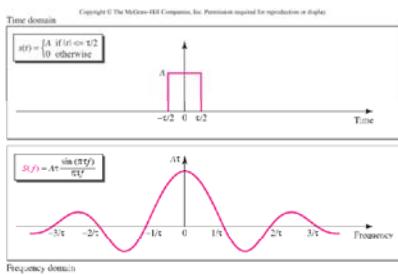


a. Time domain



b. Frequency domain

### Example of a Fourier Transform



### Inverse Fourier Transform

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$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi ft} dt$$

Fourier transform

$$s(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi ft} dt$$

Inverse Fourier transform

Fluorophore	Absorption $\lambda_{max}$ (nm)	Fluorescence $\lambda_{max}$ (nm)
Fluorescein	495	515
Tetramethylrhodamine	525	575

A protein uses ATP to convert CO to CO<sub>2</sub>. A pair of fluorophores (fluorescein and tetramethylrhodamine) has been chemically attached to the ATP reaction sites.

Three labeled protein samples are prepared:

**Sample I** with fluorescein attached to the ATP binding site

**Sample II** with tetramethylrhodamine attached to the CO/CO<sub>2</sub> reaction site

**Sample III** with both fluorophores attached to both the ATP and reaction sites, respectively.

Their quantum yields are measured as follows:  $\Phi_I = 0.38$ ,  $\Phi_{II} = 0.33$ , and  $\Phi_{III} = 0.22$ . *R<sub>0</sub> for the fluorescein –tetramethylrhodamine pair is 55 Å.*

- Of this pair, which is the donor and which is the acceptor? Why?
- In **Sample III**, which wavelength is used to excite the fluorophore for a FRET experiment, and which wavelength is used to monitor the fluorescence?
- What is the energy transfer efficiency (*E<sub>ff</sub>*)?
- What is the spatial separation (in Å) between the binding and reaction sites?